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THE COMPETITION BETWEEN NUCLEAR SHELLS IN MAYER'S MODEL †

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Received 3 April 1956

Abstract: Some regularities in nuclear shell filling and in nuclear energy level systematics are explained in Mayer's single-particle model.

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Part I: Introductory

1. PURPOSE AND PLAN OF THE WORK

Nuclear shell filling, as examplified in Klinkenberg's tables [1]), appears at first sight quite irregular. It is the purpose of this paper to see if there is regularity in this apparent disorder, and if we can understand it more definitely than by the qualitative statement of Mayer [2]), which says that pairs of spin levels arising from adjacent orbital levels can, and often will, cross as an effect of spin-orbit coupling.

The configurations of even-mass nuclei must usually be inferred from those of the odd-mass ones. One has, therefore, to begin the study with the odd-mass nuclei.

When trying to assign a nuclear configuration in an odd-mass nucleus, the spin and the magnetic moment enable one only to

determine the shell of the odd particle, but not how many particles it contains and not how the particles of the even kind are distributed among their shells. The quadrupole moment may answer the first question, but for large j's its answers are unreliable, and it gives no information whatsoever on the configuration of the even group of nucleons. It should also be noted that in the strict single-particle model the quadrupole moment of odd-neutron nuclei should vanish; but the deformable-core model [3]) enables one to make use of odd-neutron quadrupoles, too.

On the other hand, a quantitative treatment of shell filling should be based on energy considerations and should, therefore, assume the configurations of the competing states. As the knowledge of the moments does not suffice to attain this, one has to make use also of the systematic variations of the energies of the states when passing from one nucleus to another within the same shell [4]). In this way one comes to consider nuclear shell filling and energy systematics simultaneously.

The lighter nuclei where neutrons and protons fill the same shells and also later the $1f_{7/2}$ shell, being filled regularly with the possible exception [1]) of F^{19}, are not considered. Furthermore, the consideration is restricted to the framework of Mayer's original model, thus excluding the $(g^n_{9/2})_{7/2}$ states which compete with the $1g_{9/2}$ and $2p_{1/2}$ states on the ground level. Likewise, we do not consider the rare earth region, where the large quadrupole moments of the ground states [5]) and the interval ratios between excited levels [6]) show that the lower states are not single-particle states at all. Finally, for want of sufficient experimental data, we do not consider nuclei containing more than 82 protons or 126 neutrons. Practically speaking, the treatment is confined to the conventional "islands of isomerism", which formed one of the earliest supports of the single-particle model.

After the introduction, the work is divided into two parts. In the first of these, energy formulas are calculated (when not already available), summarized and discussed, and in the second part the results are applied to nuclear shell filling proper.

2. THE APPROXIMATIONS ASSUMED (DESCRIPTIVE)

At the basis of the shell model lies the central field with a strong spin-orbit interaction zero-order approximation, in which a nuclear state is described by the quartets of $nljm$ quantum numbers

defining the states of the individual nucleons. The energy of the nucleus is the sum of the energies of the individual nucleons and depends, therefore, only on the group of the nlj's — the jj-configuration — and is very highly degenerate. The central field assumed for nucleons is something between a spherical well and a harmonic-oscillator potential, and the spin-orbit parameter is assumed to be negative. The order of single particle levels in this approximation is roughly given in table II of Mayer [2]), reproduced here (table 1) for easier orientation.

For lighter nuclei, in which the Coulomb forces are negligible as compared to the charge-independent nuclear forces, the central field is the same for neutrons and for protons. Then there is a further degeneracy of the energy, due to all zero-order states obtained from one another by permuting neutrons with protons having the same energy. Therefore, in these nuclei one cannot usually speak of the states of the neutrons and protons separately in a given state of the nucleus.

On the other hand, in heavier nuclei the increasing Coulomb interaction makes the central field different for neutrons and for protons, which is reflected in the fact that the neutrons and the protons are filling different shells in the same nucleus. Therefore, the above degeneracy does not exist, for one cannot exchange a proton for a neutron, as all the proton states are already occupied by neutrons in the closed shells. Therefore, the nuclear state will really be a product of a proton state by a neutron state, and one may sensibly speak of the neutron and the proton configurations in a given nuclear state, as did Mayer [2]).

In the next approximation the remainder of the internucleonic interaction is treated as a perturbation (TAS [7]), chapter 6), which removes part of the degeneracy and splits the configuration into states with different energies and J-values. In general, the J-values are not sufficient for the characterization of the different levels and further quantum numbers are needed. In some configurations we need only the coupling order (TAS [7]), 1[8]), but for configurations containing many equivalent nucleons such a characterization is impossible, and we need other kinds of quantum numbers. One of these may be the seniority number, introduced by Racah [8]) into atomic spectroscopy. In the heavier nuclei we shall discuss, where the neutrons and the protons are found separately in their respective antisymmetric states, this number may be used in the

TABLE 1

Order of Energy Levels Obtained from Those of a Square Well Potential by Spin-Orbit Coupling [2])

Osc. No.	Square Well	Spin Term	No. of States	Sheils	Total No.
0	1s	$1s_{1/2}$	2	2	2
1	1p	$1p_{3/2}$	4	6	
		$1p_{1/2}$	2		8
2	1d	$1d_{5/2}$	6	12	
		$1d_{3/2}$	4		
	2s	$2s_{1/2}$	2		20
		$1f_{7/2}$	8	8	28
3	1f	$1f_{5/2}$	6	22	
	2p	$2p_{3/2}$	4		
		$2p_{1/2}$	2		
		$1g_{9/2}$	10		50
4	1g	$1g_{7/2}$	8	32	
	2d	$2d_{5/2}$	6		
		$2d_{3/2}$	4		
	3s	$3s_{1/2}$	2		
		$1h_{11/2}$	12		82
5	1h	$1h_{9/2}$	10	44	
	2f	$2f_{7/2}$	8		
		$2f_{5/2}$	6		
	3p	$3p_{3/2}$	4		
		$3p_{1/2}$	2		
		$1i_{13/2}$	14		128
6	1i	$1i_{11/2}$			
	2g				
	3d				
	4s				

same way as in atoms. The simplest way to introduce it conceptually is the following:

The antisymmetric J-states of a j^n configuration may be divided into states obtainable from those of j^{n-2} by adding to them a pair of nucleons coupled into a $(J = 0)$-state and antisymmetrizing, and states unobtainable in this way. If one starts in the above mentioned way to build the states of j^n from those of j for odd n, and from those of j^2 for even n, each J-state of j^n may be assigned a seniority number v, determining the configuration j^v in which this J-state could not be obtained from j^{v-2} by adding a pair $j^2(0)$ and antisymmetrizing, but from which onwards it could be obtained by such a process. $\frac{1}{2}(n-v)$ measures, in a certain sense, the number of pairs $j^2(0)$ in a given J-state of seniority v.

Mayer's [9]) original explicit calculations, translated into the seniority language, show that for attractive short-range forces between the nucleons, the lowest states will be those of lowest seniority number. This result was later obtained in various other ways for various cases [10, 11, 12, 13]). From it follow the postulates of Mayer for her single-particle model, and this will also be the approximation we shall adopt in the following: the states $J = 0$ of the even group and $J = j$ of the odd group postulated by Mayer [2]) are those of seniority 0 and 1, respectively. The first excited J-states of even nuclei should have seniority 2 according to the same calculations [11, 13]): a pair of nucleons "opens" from a $j^2(0)$ to a $j^2(J)$ state, and all other pairs remain coupled in $j^2(0)$ states (disregarding the antisymmetrization, of course).

3. Limitation to short-range forces and to constant parameters

Contrary to the case of atomic spectroscopy, the number of levels known in one nucleus is much smaller than the number of parameters entering into the energy formulas, and tests of the theory by least-square calculations are impossible.

A step towards an escape from this difficulty is the use of the same parameters in neighbouring nuclei. The method justified itself empirically in atoms [14]) and may also be tried with nuclei. Moreover this is a reasonable assumption for nuclei, as owing to the short range and the saturation properties of the nuclear forces, the average binding energy per nucleon is approximately constant throughout the periodic chart of nuclei. The nuclear density, too, is approxima-

tely constant (as reflected by the $A^{1/3}$ dependence of the nuclear radius), and therefore there is no apparent reason to assume a change of the nuclear Slater parameters with the numbers of the nucleons†.

This in itself will not suffice. An additional simplification may be obtained by the use of delta-forces, as here the direct and the exchange parameters are equal, and each F^k equals $(2k+1)F^0$, so that the number of independent parameters is very much reduced. As the experimentally suggested postulates at the basis of the model are theoretically justified only for delta-forces, it may be hoped that the use of these forces will give a fairly approximate representation of the facts [9, 15, 16, 17])††.

We shall base the following work on the above two assumptions, as they suffice to make the number of parameters less than the number of levels.

Part II: Theoretical

CHAPTER 1. ENERGY CALCULATIONS

4. The energy expression in a nuclear state

As we limit ourselves to first-order perturbation theory, the value of the energy is the sum of the values of the interactions between all pairs of the nucleons. Consequently we may write up to the first order

$$E = E_0+\Sigma n_i a_i+\Sigma p_k A_k+W(j_i^{n_i} j_k^{p_k}) \qquad (1)$$

where E denotes the binding energy of the nuclear state $(j_i^{n_i} j_k^{p_k})$ in the first approximation, E_0 is the energy of the closed shells, a_i the sum of the interaction of a single neutron in the shell j_i with the closed shells and of its spin-orbit energy, A_k the same for a single proton in the shell j_k, n_i and p_k the numbers of the neutrons in the shell j_i and of the protons in the shell j_k, respectively, and W is the interaction energy of the nucleons in the open shells.

For the sake of simplicity we shall assume that there are two incomplete neutron shells, j_1 and j_2, and two incomplete proton shells, j_3 and j_4. Then

† *Note added in proof:* Recent detailed calculations carried out at Rehovoth[102]) confirm the legitimacy of assuming a nuclear field which is approximately constant in neighbouring nuclei.

†† See in this connection the note added at the end.

$$W(j_1^{n_1}j_2^{n_2}j_3^{p_3}j_4^{p_4}) = W(j_1^{n_1})+W(j_2^{n_2})+W(j_3^{n_3})+W(j_4^{n_4}) \\ +I(j_1^{n_1}j_2^{n_2})+I(j_3^{p_3}j_4^{p_4}) \\ +I(j_1^{n_1}j_3^{p_3})+I(j_1^{n_1}j_4^{p_4})+I(j_2^{n_2}j_3^{p_3})+I(j_2^{n_2}j_4^{p_4}), \tag{2}$$

where each W denotes the internal energy of the configuration in the brackets and each I denotes the interaction between the two groups of nucleons in the two shells within its brackets. We shall have to calculate the expressions appearing in the three lines of (2).

For the sake of simplicity we shall restrict ourselves to central and charge-independent two-body interactions. These are linear combinations of Wigner, Majorana, Bartlett and Heisenberg interactions (*Nuclear Forces* [18]), equation 8.2—4). Moreover, since we are working with delta forces, Majorana forces give the same results as Wigner forces, and Heisenberg's are the same as Bartlett's, which makes their calculation superfluous. Also, for nucleons of the same kind, Heisenberg interaction equals Wigner's with the sign reversed and Bartlett's equals Majorana's with the sign reversed. Therefore, for delta-forces and nuclei of the same kind, only Wigner energy has to be calculated.

5. The energy of a group of equivalent nucleons of the same kind

For delta-forces the seniority is a good quantum number, and one has [19])

$$W^w(j^n{}_vJ) = W^w(j^v{}_vJ)+\tfrac{1}{2}(n-v)W^w(j^2(0)) \tag{3}$$

(where W^w denotes Wigner interaction), and therefore one has for an even group in its ground state of seniority 0

$$W^w(j^{2n}(0)) = n\cdot W^w(j^2(0)), \tag{4a}$$

for an odd group in its ground state of seniority 1

$$W^w(j^{2n+1}(j)) = n\cdot W^w(j^2(0)), \tag{4b}$$

and for an even group in an excited state of seniority 2

$$W^w(j^{2n}(J)) = W^w(j^2(J)) + (n-1)\cdot W^w(j^2(0)). \tag{4c}$$

The values of $W^w(j^2(J))$ were calculated by Talmi [16]). His results may be written in the form

$$W^w(j^2(J)) = \frac{(2j+1+J)(2j-J)!J!^2(j-\frac{1}{2}+\frac{1}{2}J)!^2}{2(2j+J)!(\frac{1}{2}J)!^4(j-\frac{1}{2}-\frac{1}{2}J)!^2}\cdot F^0 \tag{5}$$

which gives

$$W^w(j^2(0)) = \frac{2j+1}{2}\cdot F^0 \tag{5a}$$

$$W^w(j^2(2)) = \tfrac{1}{8} \cdot \frac{(2j-1)(2j+1)(2j+3)}{2j(2j+2)} \cdot F^0 \qquad (5b)$$

$$W^w(j^2(4)) = \frac{9}{128} \cdot \frac{(2j-3)(2j-1)(2j+1)(2j+3)(2j+5)}{(2j-2)2j(2j+2)(2j+4)} \cdot F^0 \qquad (5c)$$

for the low-lying states we shall consider.

6. The interaction between nucleons of the same kind in two different shells

Here, too, it is sufficient to calculate Wigner interaction. As at least one of the two shells is in a $(J = 0)$-state, only the scalars will contribute to the interaction, and therefore

$$I^w(j_1^{n_1} j_2^{n_2}) = n_1 \cdot n_2 \cdot I_0^w(j_1 j_2) \qquad (6)$$

where $I_0(j_1 j_2)$ is the scalar part of the interaction $I(j_1 j_2)$ between the two nucleons.

The scalar part of the direct integral is F^0. The spatial part of the exchange integral equals the direct integral in the case of delta-forces. Therefore, the exchange energy is the same as if the coefficient of each G^k were

$$-(\tfrac{1}{2}+2(\mathbf{s}_1 \cdot \mathbf{s}_2))(\mathbf{c}_1^{(k)} \cdot \mathbf{c}_2^{(k)}) = -\tfrac{1}{2}(\mathbf{c}_1^{(k)} \cdot \mathbf{c}_2^{(k)}) - 2(\mathbf{s}_1 \cdot \mathbf{s}_2)(\mathbf{c}_1^{(k)} \cdot \mathbf{c}_2^{(k)}) \qquad (7)$$

and from this only the scalars have to be taken into account. From the first term one gets $-\frac{1}{2}F^0$, and the second term vanishes, as only $\mathbf{c}^{(1)}$ can produce a scalar with the vector $\mathbf{s}$, when their product is decomposed with respect to $\mathbf{J}$. But $\mathbf{c}^{(1)}$ vanishes according to (II—51) [20]. The final expression is

$$I^w(j_1^{n_1} j_2^{n_2}) = \tfrac{1}{2} \cdot n_1 \cdot n_2 \cdot F_{12}^0. \qquad (8)$$

7. The interaction between protons and neutrons

Here, one has to calculate both Wigner and Bartlett interactions, but there is only a direct integral in our heavy nuclei. Equation (6) holds here mutatis mutandis:

$$I(j_1^{n_1} j_3^{p_3}) = n_1 \cdot p_3 \cdot I_0(j_1 j_3). \qquad (9)$$

With Wigner forces one gets the same result as with the direct integral of the preceding section [21]:

$$I^w(j_1^{n_1} j_3^{p_3}) = n_1 \cdot p_3 \cdot F_{13}^0. \qquad (10)$$

For Bartlett interaction the coefficient of F^k is

$$(\tfrac{1}{2}+2(\mathbf{s}_1 \cdot \mathbf{s}_2))(\mathbf{c}_1^{(k)} \cdot \mathbf{c}_2^{(k)})$$

and exactly as in the previous section only the coefficient $\frac{1}{2}$ of F^0 will be different from zero. The result is (with I^b meaning Bartlett interaction)

$$I^b(j_1^{n_1} j_3^{p_3}) = \tfrac{1}{2} \cdot n_1 \cdot p_3 \cdot F^0_{13}. \tag{11}$$

8. Configuration-interaction elements

In our approximation (see also ref. [21]) an even group of neutrons or protons in an odd-mass nucleus or in the ground state of an even nucleus is divided into a sum of even groups, each in a state of seniority 0 within its own shell. An odd group has one odd shell in a state of seniority 1 and the other shells are even in states of seniority 0. In an even nucleus in a low excited state one of the shells is in a state of seniority 2 and the others are in states of seniority 0. The interaction matrices between low-lying states will therefore have non-vanishing elements only between configurations in which one pair of $j^2(0)$-nucleons lies in different shells. Only the interaction of that kind of nucleons, whose configuration is changed from state to state, will contribute to the interaction element between the states. Therefore, it will suffice to calculate non-diagonal elements only for Wigner forces according to the remarks made at the end of section 4.

We have to calculate matrix elements of the form

$$(j_1^{n_1}(J),\, j_2^{n_2}(0)\, |\Sigma V_{ik}| j_1^{n_1-2}(J),\, j_2^{n_2+2}(0)),$$

where the $J = 0$ is always of seniority 0, and J may equal 0 of seniority 0, j of seniority 1 or J' of seniority 2, according to the type of the interacting states. It is difficult to obtain a closed formula for the matrix element for general n_1 and n_2. However, in this work we shall only need the element for $n_2 = 0$, for which case it is given by the jj-analogue of (III—33c) [8]), which reads in the present case:

$$\begin{aligned}&(j_1^n(J)\, |\Sigma V_{ik}| j_1^{n-2}(J),\, j_2^2(0),\, J)\\ &\quad = (\tfrac{1}{2}n(n-1))^{1/2}\, (j_1^n(J)\{|j_1^{n-2}(J),\, j_2^2(0),\, J)\, (j_1^2(0)\, |V_{12}| j_2^2(0)).\end{aligned} \tag{12}$$

The two particle matrix element is most readily calculated by transforming from jj to LS scheme, the coefficients of which transformation were calculated by Racah [22]). For delta-forces the interaction differs from zero only between the space-symmetrical singlet states, therefore

$$(j_1^2(0)|V_{12}^w|j_2^2(0)) = (j_1^2(0)|^1S(0))\,(l_1^2\,{}^1S|V_{12}^w|l_2^2\,{}^1S)\,(^1S(0)|j_2^2(0))$$
$$= \left[\frac{2j_1+1}{2(2l_1+1)}\right]^{1/2} \cdot (l_1^2\,{}^1S|V_{12}^w|l_2^2\,{}^1S) \cdot \left[\frac{2j_2+1}{2(2l_2+1)}\right]^{1/2} \quad (13)$$
$$= (-1)^{l_1+l_2} \cdot \tfrac{1}{2}((2j_1+1)(2j_2+1))^{1/2} \cdot F_{12}^0.$$

The final result was arrived at by using equations (38), (51), (50) and (36′) of II [20]) to obtain the value of the coefficient of F_{12}^k (to which the non-diagonal radial integral is equal in the case of delta-forces), and then inserting $F^k = (2k+1)F^0$ and summing over k according to (II−20b) [20]).

The coefficients of fractional parentage in (12) were calculated by Schwartz and de Shalit [23]):

$$(j^n(0)\{|j^{n-2}(0),\ j^2(0),\ 0) = \left[\frac{2j+3-n}{(n-1)(2j+1)}\right]^{1/2}, \quad (14a)$$

$$(j^n(j)\{|j^{n-2}(j),\ j^2(0),\ j) = \left[\frac{2j+2-n}{n(2j+1)}\right]^{1/2}, \quad (14b)$$

and

$$(j^n(J')\{|j^{n-2}(J'),\ j^2(0),\ J') = \left[\frac{(n-2)(2j+1-n)}{n(n-1)(2j+1)}\right]^{1/2}. \quad (14c)$$

Inserting the results of (13) and (14, a, b, c) in (12), one obtains for the interaction element of an even group in its ground state

$$(j_1^{2n_1}(0)|\Sigma V_{ik}^w|j_1^{2n_1-2}(0),\ j_2^2(0),\ 0)$$
$$= (-1)^{l_1+l_2} \cdot (2n_1\,(2j_1+3-2n_1)(2j_2+1)/8)^{1/2} \cdot F_{12}^0, \quad (15a)$$

for that of an odd-group

$$(j_1^{2n_1+1}(j)|\Sigma V_{ik}^w|j_1^{2n_1-1}(j),\ j_2^2(0),\ j)$$
$$= (-1)^{l_1+l_2} \cdot (2n_1(2j_1+1-2n_1)(2j_2+1)/8)^{1/2} \cdot F_{12}^0, \quad (15b)$$

and for that of an even group in an excited J'-state of seniority 2

$$(j_1^{2n_1}(J')|\Sigma V_{ik}^w|j_1^{2n_1-2}(J'),\ j_2^2(0),\ J')$$
$$= (-1)^{l_1+l_2} \cdot ((2n_1-2)(2j_1+1-2n_1)(2j_2+1)/8)^{1/2} \cdot F_{12}^0. \quad (15c)$$

CHAPTER 2

A PRELIMINARY DISCUSSION AND PREPARATIONS

9. The factors governing the order of filling

With a delta-type interaction there are only Wigner and Bartlett central interactions. As long as the coefficient of the latter in the

mixture is less than $\frac{1}{2}$, the energy formulas will be the same as for pure Wigner interaction, only with a different definition (which, however, preserves the sign) of the parameters. From the treatment of the saturation properties of nuclear forces (*Nuclear Forces* [18]), eq. 11.3—11), as well as from a study of the ground states of odd nuclei [17, 24]), is is to be concluded that this is the actual situation with nuclei, and, therefore, we shall use only the Wigner forces formulas (5), (8), (10) and (15).

Denoting by π the pairing energy (5a) for neutrons and by Π the same for protons, and using the formulas of the preceding chapter, eq. (1) may be written:

$$\begin{aligned} E(j_1^{2n_1} j_2^{2n_2} j_3^{2p_3} j_4^{2p_4}) &= E_0+2n_1 a_1+2n_2 a_2+2p_3 A_3+2p_4 A_4 \\ &+n_1\pi_1+n_2\pi_2+p_3\Pi_3+p_4\Pi_4 \\ &+\tfrac{1}{2}\cdot 2n_1\cdot 2n_2\cdot F_{12}^0+\tfrac{1}{2}\cdot 2p_3\cdot 2p_4\cdot F_{34}^0 \\ &+2n_1\cdot 2p_3\cdot F_{13}^0+2n_1\cdot 2p_4\cdot F_{14}^0+2n_2\cdot 2p_3\cdot F_{23}^0+2n_2\cdot 2p_4\cdot F_{24}^0, \end{aligned} \tag{16}$$

for even nuclei in their ground states. The formula for odd-mass nuclei in the ground state is obtained from (16) by replacing $2n_1$ by $(2n_1+1)$ in the first, third and fourth lines, for an odd-j_1-neutron nucleus, for example.

In order to see how the neutrons will arrange themselves for a given proton configuration, (16) may be re-written in the form

$$\begin{aligned} &E(j_1^{2n_1} j_2^{2n_2} j_3^{2p_3} j_4^{2p_4}) \\ &= E_0+2p_3 A_3+2p_4 A_4+p_3\Pi_3+p_4\Pi_4+\tfrac{1}{2}\cdot 2p_3\cdot 2p_4\cdot F_{34}^0 \qquad (17a) \\ &+2n_1(a_1+\tfrac{1}{2}\pi_1+2p_3 F_{13}^0+2p_4 F_{14}^0)+2n_2(a_2+\tfrac{1}{2}\pi_2+2p_3 F_{23}^0+2p_4 F_{24}^0) \\ &+\tfrac{1}{2}\cdot 2n_1\cdot 2n_2\cdot F_{12}^0, \end{aligned}$$

and one sees that, were it not for the last term, all the neutrons would have gone to the shell j_1 or to shell j_2, depending on the coefficient of $2n_1$ in the second line being larger or smaller than that of $2n_2$. On the other hand, the interaction between the two neutron shells tends to distribute the neutrons equally between them, and if the coefficient of $2n_1$ nearly equals that of $2n_2$, this interaction will actually have a decisive influence on the filling-order of the shells. This is a special property of short-range forces, due to the fact that for them the internal energy of each shell is a linear function of the number of nucleons in the shell, as owing to the short range combined with the Pauli principle, each nucleon interacts, on the average, only with its partner in the $j_2^2(0)$ state (formula (4a)). With longer-

range forces each nucleon interacts with all its equivalents, the shell energy contains a term proportional to $n(n-1)/2$, and the factors tending to concentrate all the nucleons in one shell are much stronger.

For odd-mass nuclei one may write

$$\begin{aligned} &E(j_1^{2n_1+1} j_2^{2n_2} j_3^{2p_3} j_4^{2p_4}) \\ &= E_0+2p_3 A_3+2p_4 A_4+p_3 \Pi_3+p_4 \Pi_4+\tfrac{1}{2}\cdot 2p_3\cdot 2p_4 F^0_{34} \\ &\quad +2n_1(a_1+\tfrac{1}{2}\pi_1+2p_3 F^0_{13}+2p_4 F^0_{14})+2n_2(a_2+\tfrac{1}{2}\pi_2+2p_3 F^0_{23}+2p_4 F^0_{24}) \\ &\quad +\tfrac{1}{2}\cdot(2n_1+1)\cdot 2n_2\cdot F^0_{12} \qquad (17b) \\ &\quad +a_1+2p_3 F^0_{13}+2p_4 F^0_{14}, \end{aligned}$$

and it is seen that all the factors existing in even nuclei operate here too, with the additional possibility (ignoring the $\frac{1}{2}(2n_1+1)2n_2F^0_{12}$ term) that all the pairs of the odd group may go to shell j_2, for example, in case the coefficient of $2n_2$ is larger than that of $2n_1$, and yet the odd nucleon will go into j_1, provided the expression written in the last line of (17b) is greater for j_1 than for j_2. This may, of course, only happen if $\pi_2 > \pi_1$.

Actually, the situation is more complex, as the proton configuration is not given ab initio, and the real nuclear configuration should be most stable not only with respect to all configurations obtained from it by keeping the proton configuration fixed and varying the neutron configuration, and vice versa, but also with respect to all configurations obtained from it by simultaneously changing proton and neutron configurations.

Up to now we have been considering a fixed nucleus. When proceeding from the sample nucleus to its isotones by adding pairs of protons always in the same shell, the equilibrium distribution of the neutrons will vary in the direction of the neutron shell preferred by the proton shell being filled. (When the interaction of the protons in shell j_3 is stronger with the neutrons in shell j_1 than with those in shell j_2 — $F^0_{13} > F^0_{23}$ — we shall say that the proton shell j_3 prefers the neutron shell j_1 over j_2). This is one aspect of the stabilization effect of de Shalit and Goldhaber [21]). (See also the following section).

When passing from the sample nucleus to its isotopes by adding neutron pairs in the same shell, the probability that the next pair will also enter the same shell continually decreases, due to the entailed rising loss of interaction energy between the two neutron shells.

10. EXCITED STATES IN ODD-MASS NUCLEI

The energy differences between two single-particle levels obtained from one another by a transition of a nucleon or of a "hole" from the odd shell to the even one are given respectively by the formulas

$$E(j_1^{2n_1+1} j_2^{2n_2} j_3^{2p_3} j_4^{2p_4}) - E(j_1^{2n_1} j_2^{2n_2+1} j_3^{2p_3} j_4^{2p_4})$$
$$= (a_1-a_2)-(n_1-n_2)F_{12}^0+2p_3(F_{13}^0-F_{23}^0)+2p_4(F_{14}^0-F_{24}^0), \quad (18a)$$

$$E(j_1^{2n_1+1} j_2^{2n_2} j_3^{2p_3} j_4^{2p_4}) - E(j_1^{2n_1+2} j_2^{2n_2-1} j_3^{2p_3} j_4^{2p_4})$$
$$= -(a_1-a_2)-(\pi_1-\pi_2)+(n_1-n_2+1)F_{12}^0 \quad (18b)$$
$$-2p_3(F_{13}^0-F_{23}^0)-2p_4(F_{14}^0-F_{24}^0).$$

The formulas are linear functions of the numbers of the nucleons in the shells, and in so far as in the progress from one nucleus to another the pairs of nucleons of one kind are always entering into the same shell and the levels are obtained from each other by a particle transition in the same direction, the graph of the energy difference between two single-particle levels in the different isotopes of an element or in the different isotones among neighbouring elements, should be a straight line†.

What determines the slopes of the linear graphs or the changes in the excitation energies? Let us consider, for instance, an excited state formed by the transition of the odd neutron from shell j_1 to shell j_2. Then, (18a) tells us that if from nucleus to nucleus the protons are added in shell j_3 which prefers j_1 to j_2, it will be harder to excite the state as the protons are added, and vice versa. This is again the stabilization effect of de Shalit and Goldhaber [21]). When neutrons are added, the excitation will be more difficult if they are added in shell j_2, since then the loss in the interaction energy between shells j_1 and j_2 is steadily increasing.

It is worth mentioning that the stabilization effect may also be produced by particles of the same kind. If there is a third even neutron shell, $j_0^{2n_0}$, one has to add to (18a) the expression $+n_0(F_{01}^0-F_{02}^0)$. This shows that an additional shell of the odd group, which does not participate in the transition forming one excited state from the other, has the same influence on the systematics as the even-group shells, but reduced by a factor of $\frac{1}{2}$ owing to the exchange energy. An example of this effect is probably the stabilization of the neutron shell $2d_{3/2}$ as compared to $3s_{1/2}$, caused by the pairs of neutrons added in $1h_{11/2}$ (section 23).

† See note added at the end.

11. Excited states in even nuclei

If we interpret the first excited state 2^+ in even nuclei as one of seniority 2, we find that the interval $E(0^+)-E(2^+)$ should be the same in neighbouring nuclei as long as the pair $j^2(0)$ is "opened" to a $j^2(2)$ state in the same shell, because the difference $W(j_1^{2n}(0))-W(j_1^{2n}(2))$ is independent of n according to (4a, c), and the interaction of j_1^{2n} with the other shells, or the interaction of the other shells among themselves, is altogether independent of the J of the j_1^{2n} shell. In this case

$$E(0^+)-E(2^+) = W(j_1^2(0))-W(j_1^2(2)). \tag{19}$$

The interval will change only when a pair will "open" in another shell and especially, therefore, when a shell with a smaller interval (19) begins to the filled with pairs.

A second possibility for a first excited state is 0^+, in case it is easier to transfer a pair $j^2(0)$ from one shell to another than to "open" a $j^2(0)$ pair, or when all the shells are filled. In particular, this is the only possibility predicted by our approximation for a double-magic nucleus. However, recent work by Elliott [25, 26]) shows that this approximation is invalid in the double-magic nucleus Pb^{208}, and according to Alburger and Pryce [27]) the same applies to Pb^{206}, too.

The second excited state of an even nucleus will be 4^+ if the pair continues to "open", and its excitation energy will be

$$E(0^+)-E(4^+) = W(j_1^2(0))-W(j_1^2(4)). \tag{20}$$

However, this will not apply if the shell has $j \leqq 3/2$, and also if it is easier to excite a 2^+-state in another shell, or to transfer a pair in a $j^2(0)$-state from shell to shell, in which cases the second excited state will be 2^+ or 0^+ respectively. Our approximation does not predict other second-excited J and parity values.

12. Consistency considerations

Most simply a single-particle level of given J may usually be obtained from another of different J in two ways, according to the particle or the "hole" making the transition from shell to shell, and therefore even linear systematics may generally be interpreted in two ways and cannot unequivocally determine the configurations. One must have recourse to considerations which will ensure that the filling order assumed is consistent.

Assigning a certain ground state configuration is equivalent to the assumption that this configuration is more stable than all other possible configurations. Similarly, assuming a configuration for a low-lying excited state of a given J different from the ground-state-J, is equivalent to the assumption that this configuration is more stable than all the other configurations with the same J-value (obtained from it by transferring $j^2(0)$-pairs from shell to shell). Such assumptions give a set of inequalities which the F^0's of each nucleus have to satisfy, and one has to impose the condition that the system of sets obtained from the various nuclei, in which the same shells are being filled, will constitute a consistent system. Furthermore, when the number of known levels suffices to obtain the values of the F^0's experimentally, the inequalities should actually be satisfied by the experimental parameters, or the interpretation of the systematics would be inconsistent even though the sets of inequalities in themselves form a consistent system.

It should be remarked that when there is an interaction between the different states, the consistency requirements are much restricted in the sense that one may compare in our approximation only configurations with the same J, as the lowest of these will also remain the lowest after the operation of the interaction, while states with different J's may be lowered by the interaction to different extents, reversing the order of their unperturbed binding energies.

13. The parameters in a harmonic-oscillator central field

In order to gain an idea of the magnitude relationships between the various F^0's, we calculated the F^0's for the case of delta-forces and a harmonic-oscillator central potential, so as to know whether the values of the parameters obtained from the systematics are plausible, and also to enable us to choose between several consistent interpretations. With delta-forces

$$V_{12} = 4\pi g \delta(\mathbf{r}_1 - \mathbf{r}_2), \tag{21}$$

the F^0's are equal to

$$F^0_{12} = F^0(n_1 l_1, n_2 l_2) = g \int_0^\infty (R_{n_1 l_1}(r))^2 (R_{n_2 l_2}(r))^2 \, dr/r^2 \tag{22}$$

A detailed account of the radial oscillator-wave-functions is to be found in the thesis of Talmi [10]). We calculated the F^0's for each (n_1, n_2) as a function of l_1 and l_2 by direct integration. However,

TABLE 2a

The F^0's for Delta-Forces and Harmonic-Oscillator Wave-Functions in Units of $g(2\nu^3/\pi)^{1/3}$: Simple Fractions

Shells	1s	1p	1d	1f	1g	1h	1i	2s	2p	2d	2f	2g	3s	3p	3d
1s	1	1/2	1/4	1/8	1/16	1/32	1/64	5/8	7/16	9/32	11/64	13/128	63/128	99/256	143/512
1p		5/12	7/24	3/16	11/96	13/192	5/128	11/48	23/96	43/192	71/384	107/768	133/768	93/512	187/1024
1d			21/80	33/160	143/960	13/1[illegible]	17/256	5/32	133/960	93/640	187/1280	1027/7680	283/2560	1609/15360	223/2048
1f				429/2240	143/896	221/1792	323/3584	9/64	69/640	891/8960	1859/17920	767/7168	87/1024	5571/71680	10769/143360
1g					2431/16128	4199/32256	323/3072	47/384	77/768	4433/53760	715/9216	10387/129024	445/6144	1837/28672	51623/860160
1h						4199/33792	7429/67584	25/256	143/1536	559/7168	2873/43008	180557/2838528	279/4096	1391/24576	17719/344064
1i							185725/1757184	37/512	415/5120	153/2048	5491/86016	15181/270336	543/8192	879/16384	4573/98304
2s								41/64	109/384	29/256	35/512	211/3072	507/1024	1825/6144	587/4096
2p									363/1280	1393/7680	1457/15360	111/2048	1091/6144	12433/61440	7169/40960
2d										6579/35840	275/2048	34957/430080	2419/20480	19079/172032	71877/573440
2f											58487/430080	55133/516096	861/8192	145223/1720320	279169/3440640
2g												136357/1261568	3929/49152	54023/688128	453947/6881280
3s													8257/16384	20591/98304	26709/327680
3p														224081/983040	632787/4587520
3d															4113433/27525120

TABLE 2b

The F^0's for Delta-Forces and Harmonic-Oscillator Wave-Functions in Units of $g(2\nu^3/\pi)^{1/2}$: Decimal Fractions

Shells	1s	1p	1d	1f	1g	1h	1i	2s	2p	2d	2f	2g	3s	3p	3d
1s	1	0.500	0.250	0.125	0.063	0.031	0.016	0.625	0.438	0.281	0.172	0.102	0.492	0.387	0.279
1p		0.417	0.292	0.188	0.115	0.068	0.039	0.229	0.240	0.224	0.185	0.139	0.173	0.182	0.183
1d			0.263	0.206	0.149	0.102	0.066	0.156	0.139	0.145	0.146	0.134	0.111	0.105	0.109
1f				0.192	0.160	0.123	0.090	0.141	0.108	0.099	0.104	0.107	0.085	0.078	0.075
1g					0.151	0.130	0.105	0.122	0.100	0.082	0.078	0.081	0.072	0.064	0.060
1h						0.124	0.110	0.098	0.093	0.078	0.067	0.064	0.068	0.057	0.051
1i							0.106	0.072	0.081	0.075	0.064	0.056	0.066	0.054	0.047
2s								0.641	0.284	0.113	0.068	0.069	0.495	0.297	0.143
2p									0.284	0.181	0.095	0.054	0.178	0.202	0.175
2d										0.184	0.134	0.081	0.118	0.111	0.125
2f											0.136	0.107	0.105	0.084	0.081
2g												0.108	0.080	0.079	0.066
3s													0.504	0.209	0.082
3p														0.228	0.138
3d															0.149

for $n_1, n_2 \geqq 1$ the formulas are very cumbersome, and we give only the results in table 2a, b, in which the parameters are given in units of $g(2\nu^3/\pi)^{1/2}$, where ν is related to the oscillator energy in an nl-state † by the equation

$$E_{nl} = (l+2n+3/2)h\nu. \tag{23}$$

This unit is of the order of 2400 keV (determined from the $E(0^+)-E(2^+)$ separation [21]) of 1510 keV of Ca^{42} or from the same separation of 1300 keV of Sn^{116} by using table 3).

TABLE 3

$W(j^2J)$ for Delta-Forces and Harmonic-Oscillator Wave-Functions in Units of $g(2\nu^3/\pi)^{1/2}$

J	$1s_{1/2}$	$1p_{3/2}$	$1p_{1/2}$	$1d_{5/2}$	$1d_{3/2}$	$1f_{7/2}$	$1f_{5/2}$	$1g_{9/2}$	$1g_{7/2}$
0	1.000	0.833	0.417	0.788	0.525	0.766	0.575	0.754	0.603
2	—	0.167	—	0.180	0.105	0.182	0.131	0.183	0.144
4	—	—	—	0.075	—	0.090	0.055	0.095	0.070
$W(0)-W(2)$	—	0.667	—	0.608	0.420	0.584	0.443	0.571	0.459
$W(0)-W(4)$	—	—	—	0.713	—	0.677	0.520	0.659	0.532

J	$1h_{11/2}$	$1h_{9/2}$	$1i_{13/2}$	$1i_{11/2}$	$2s_{1/2}$	$2p_{3/2}$	$2p_{1/2}$	$2d_{5/2}$	$2d_{3/2}$
0	0.746	0.621	0.740	0.634	0.641	0.567	0.284	0.551	0.367
2	0.182	0.151	0.182	0.155	—	0.113	—	0.126	0.073
4	0.097	0.078	0.099	0.083	—	—	—	0.052	—
$W(0)-W(2)$	0.563	0.471	0.558	0.479	—	0.454	—	0.425	0.294
$W(0)-W(4)$	0.648	0.543	0.641	0.551	—	—	—	0.498	—

J	$2f_{7/2}$	$2f_{5/2}$	$2g_{9/2}$	$2g_{7/2}$	$3s_{1/2}$	$3p_{3/2}$	$3p_{1/2}$	$3d_{5/2}$	$3d_{3/2}$
0	0.544	0.408	0.540	0.432	0.504	0.456	0.228	0.448	0.299
2	0.130	0.093	0.131	0.103	—	0.091	—	0.102	0.060
4	0.064	0.039	0.068	0.051	—	—	—	0.043	—
$W(0)-W(2)$	0.414	0.315	0.409	0.329	—	0.365	—	0.346	0.239
$W(0)-W(4)$	0.480	0.369	0.472	0.382	—	—	—	0.406	—

In order to facilitate the consideration of pair filling and of excited states in even nuclei we have also given the energies of

† It should be noted that the n appearing in (22) is smaller by one than the number preceding the l in the customary designation of nuclear shells, so that $F^0(1f, 2p)$ should be written $F^0(03, 11)$ in (22), if one wants to use the standard notation of the harmonic-oscillator wave-functions [10]).

$j^2(J)$ for the different j's and $J \leqq 4$ in table 3, together with the energies of excitation of $j^2(J)$ from the ground $j^2(0)$ state.

It should be remarked that one may expect the magnitude relations of table 2 not to hold for parameters between neutron and proton shells, as in the heavy nuclei we are going to consider the central zero-order field is already not the same for protons and for neutrons.

14. The use of nuclear quadrupole moments to assign configurations

For the average value of the quadrupole moment of an odd group in a state of seniority 1, one has (as the quadrupole tensor is of even rank), in accordance with the jj-analogue of (III—69a) [8])

$$\overline{Q}(j^{2n+1}(j)) = ((2j+1-2n)/(2j-1)) \cdot \overline{Q}_{\text{s. p.}}$$
$$= -((2j+1-2n)/(2j+2)) \cdot \overline{er^2}, \quad (24)$$

where $\overline{Q}_{\text{s. p.}}$ denotes the average value of the quadrupole moment of a single nucleon in the given j-state. e is the nucleon charge and $\overline{r^2}$ the average value of the square of its distance from the centre of the field, which is very roughly equal to the square of the nuclear radius given by $R = 1.5A^{1/3} \cdot 10^{-13}$ cm. One sees that the quadrupole is a linear function of n, changing sign from negative to positive when n passes the middle of the shell.

In a mixed state

$$\psi = \alpha\psi_1 + \beta\psi_2 \qquad (\alpha^2+\beta^2 = 1) \quad (25)$$

one has in our approximation (section 8) in a self-evident notation

$$\overline{Q} = \alpha^2\overline{Q}_1 + \beta^2\overline{Q}_2 \quad (26)$$

as the interaction is between states differing in the shell by at least one pair of nucleons, and the quadrupole moment is a single-particle operator (TAS [7]), 5 [6]) whose non-diagonal elements vanish between such states.

15. Arrangement of the experimental material in the following chapters

The following chapters deal with the competitions between the nuclear shells which are responsible for the main groups of isomers before $N = 50$, $Z = 50$, $N = 82$ and $Z = 82$, $N = 126$. The same shells are being filled in both odd-Z and odd-N nuclei of about

the same mass number A, and in so far as experimental data are available, both groups are dealt with together each time.

In each chapter we give tables showing the distribution of the nucleons among the shells in the ground state, as far as we succeeded to determine it. The configurations are marked by the following letters:

a an odd-group configuration determined by the nuclear moments, or a configuration consisting of closed shells or sub-shells,

b configurations determined by a quantitative consideration of energy level systematics in odd-mass nuclei,

c configurations determined by consistency considerations,

e configurations determined by a qualitative discussion of the excited levels of even nuclei.

Configurations not designated by either of these letters are also sometimes given, but they are only tentative. Besides, the tables give the nuclear moments, if known, with references. When the spin was measured directly and not only inferred from interpretation of disintegration data, we give both the "direct" and the "disintegration" references, the first being marked (d). Radioactive nuclei are marked (*).

We also give diagrams, showing energy level systematics. For odd-mass nuclei only single-particle levels with known spin and parity values are given. The analogous levels in neighbouring isotopes of the same element are joined by a line, marked by the l_j value of the level and the symbol of the element. As abscissa we chose the mass number A, which seemed to us most appropriate to reveal the influence of the various factors on the systematics, especially the linearity in the proton numbers, which is lost sight of when choosing as abscissa the neutron number N, as in the diagrams of Goldhaber and Hill [4]). A linearity in the neutron numbers is, of course, reflected in a linearity of the lines themselves.

For even nuclei the arrangement is the same, only that here some of the levels were tentatively assigned a 2^+ value by their investigators from systematics-regularity considerations alone.

Sometimes, theoretical curves, obtained by least-square calculations given in the text, are drawn in broken lines in the diagrams, to show the agreement between theory and experiment. References for the diagrams and for the numerical values used in the calculations in the text are given with the diagrams.

Part III: Actual Nuclear Shell Filling

CHAPTER 3. THE COMPETITION BETWEEN THE NEUTRON SHELLS $2p_{1/2}$, $1g_{9/2}$ AND THE COMPETITION BETWEEN THE PROTON SHELLS $2p_{3/2}$, $1f_{5/2}$

16. The preference of the neutron shells by the proton shells, and the proton configurations of Kr^{85}, Zr^{89} and Mo^{91}

In Kr^{85}, Sr^{87}, Zr^{89} and Mo^{91}, with 49 neutrons each, the odd neutron configurations are uniquely determined by the J-values of the states (table 4). The ground state is $g_{9/2}$, and the $p_{1/2}$ state

TABLE 4

Ground States Configurations of Odd-N Nuclei before $N = 50$

Nucleus	Z	N	$I\pm$	Ref.	μ	Ref.	Q	Ref.	Proton Shells $p_{3/2}$	$f_{5/2}$	$p_{1/2}$	$g_{9/2}$		Neutron Shells $p_{1/2}$	$g_{9/2}$	
Zn^{69*}	30	39	1/2−	33					2					1		a
Ge^{71*}	32		1/2−	30					?	?				1		a
Ge^{73}	32	41	9/2+	49(d),33	−0.877	49	−0.22	49	?	?				?	?	
Se^{75*}	34		5/2 †	49(d)	−1.61	49	1.1	62	?	?				?	?	
Se^{77}	34	43	1/2−	49(d),33	0.533	49			?	?				1	4	a
Kr^{79*}	36		1/2−	32					?	?				1	4	a
Ge^{77*}	32	45	7/2+ †	33					?	?				?	?	
Se^{79*}	34		7/2+ †	49(d),33	−1.015	49	0.7	49	?	?				?	?	
Kr^{81*}	36		7/2+ †	33					?	?				?	?	
Se^{81*}	34	47	1/2−	33					?	?				1	8	a
Kr^{83}	36		9/2+	49(d),33	−0.967	49	0.16	49	2	6			b	2	7	b
Sr^{85*}	38		9/2+	33					4	6			a	2	7	b
Kr^{85*}	36	49	9/2+	63(d),33	−1.001	63	0.27	63	2	6			b	2	9	a
Sr^{87}	38		9/2+	49(d),33	−1.089	49			4	6			a	2	9	a
Zr^{89*}	40		9/2+	33					4	6	2		b	2	9	a
Mo^{91*}	42		9/2+	64					4	6	2	2	b	2	9	a

† Not a single-particle state.

is obtained by a neutron transition from $p_{1/2}$ to $g_{9/2}$. In passing from a nucleus to its heavier isotone in th above sequence it is always harder to excite the $p_{1/2}$ state (fig. 1), which shows

that the protons are added in shells preferring p_n to g_n. † An immediate corollary is that g_p prefers p_n to g_n, because in the passage from Sr^{87} to Mo^{91} via Zr^{89} at least one proton pair must enter $g_{9/2}$. This corollary shows that already in these not very heavy nuclei the central field is so different for protons and for neutrons, that the magnitude relationships between parameters pertaining to a neutron and a proton may contradict table 2.

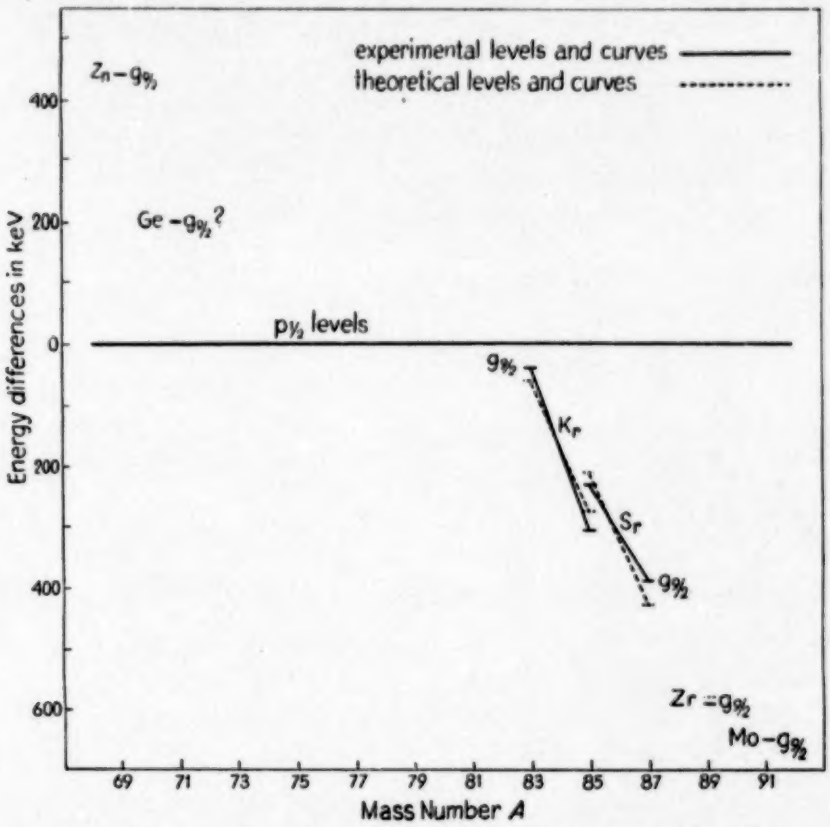

Fig. 1. $p_{1/2}$ and $g_{9/2}$ levels of odd-N nuclei before $N = 50$.
References: Zn^{69} (33); Ge^{71} (30, 31); $Kr^{83, 85}$, $Sr^{85, 87}$, Zr^{89} (33); Mo^{91} (64).

From the very high excitation energy of 913 keV of the $g_{9/2}$ state in Y^{89} (fig. 2, following chapter), it is evident that the last proton pair in Zr^{89} entered the $p_{1/2}$ proton shell, as otherwise the expression

$$(\Pi_{9/2}-\Pi_{1/2})+2(F^0(g_n p_p)-F^0(g_n g_p))$$

would have to be more than double the 913 keV, which is very

† p_n will designate a p-shell of neutrons, whether $p_{3/2}$ or $p_{1/2}$, which designation is appropriate here as the preference mentioned depends only on the l and not on the j-value of the shell according to the formulas of section 7. Likewise, g_p will designate either the $g_{9/2}$ or the $g_{7/2}$ proton shell, and a similar designation will be adopted in such contexts.

improbable both according to table 2 and to the F^0's we shall later obtain from the experimental data, whatever the sign of $F^0(g_n p_p) - F^0(g_n g_p)$. And the more so, as experimental data indicate a negative sign of this last quantity [21]).

The 20th proton pair enters $p_{1/2}$ also in the neighbouring nucleus Zr^{90}, as indicated by its first excited state 0^+ of 1750 keV [28]). Here the neutron number is magic, so the 0^+-state is probably produced by the protons, which cannot be excited to a 2^+-state, as they have only closed shells if the last pair is a $p^2_{1/2}$. The other possibility, that in the ground state of Zr^{90} the proton configuration is $g^2_{9/2}(0)$ and in the excited state $p^2_{1/2}(0)$, is even much less probable than a $g^2_{9/2}$ proton configuration in Zr^{89}, as this would imply

$$2(A_{9/2} - A_{1/2}) + (\Pi_{9/2} - \Pi_{1/2}) = 1750 \text{ keV},$$

or, substituting 913 keV for $A_{1/2} - A_{9/2}$,

$$\Pi_{9/2} - \Pi_{1/2} = 3576 \text{ keV}.$$

Ford [29]) also bases a similar argument for a $p^2_{1/2}$ proton ground configuration on the fact that up to 2 MeV there is no 2^+ excited state in Zr^{90}, whilst $E(g^2_{9/2}(0^+)) - E(g^2_{9/2}(2^+))$ cannot be so high, as is actually obvious from table 3.

From the fact that in proceeding from Sr^{87} to Zr^{89} the protons enter $p_{1/2}$, and that in this passage it becomes harder to excite the $p_{1/2}$ state, it is seen that p_p prefers p_n to g_n (which this time accords with table 2). This applies to both $p_{3/2}$ and $p_{1/2}$ protons.

The excitation energy of $p_{1/2}$ increases also from Kr^{85} to Sr^{87}, which shows that the last pair of protons in Sr^{87} entered a shell again preferring p_n to g_n, and as f_p probably prefers g_n to p_n (see below), we assume that this shell is $p_{3/2}$, and that the proton configuration of Kr^{85} is $p^2_{1/2} f^6_{5/2}$.

The conclusion that f_p prefers g_n to p_n (which agrees with table 2) is founded on the suggested decrease [30]) in the excitation energy of the $g_{9/2}$ from the $p_{1/2}$ state when passing from Zn^{69} to Ge^{71}. Here the single odd neutron (table 4) passes from the $p_{1/2}$ to the $g_{9/2}$ shell when excited, and the fact that this is easier in Ge^{71} shows that its last proton pair entered a shell preferring g_n to p_n. Such a shell can only be $f_{5/2}$, as $p_{3/2}$ was seen above to prefer p_n to g_n. However, the interpretation [30]) of the 198 keV state of Ge^{71} as $g_{9/2}$ is not uniquely agreed upon [31]), owing to contradicting values obtained for K/(L+M) ratios by the two investigating groups. A clarification of the matter would be very desirable.

In this connection it is also important to decide whether the ground state of Se^{73} is $g_{9/2}$ or $p_{1/2}$ [61].

17. The parallelogram principle

Let us consider the difference $\Delta = E(j_1)-E(j_2)$, when the state j_2 is obtained from j_1 by a transition of the odd particle from shell j_1 to shell j_2, in the four odd-N nuclei (Z, N), $(Z, N+2)$, $(Z+2, N)$ and $(Z+2, N+2)$, where the proton configuration is the same in each of the two pairs of isotopes and the neutron configuration is the same in each of the two pairs of isotones. If, for example, the last neutron pair entered in the heavier isotopes into shell j_2, and the last proton pair entered in the heavier element into shell j_3, we have from (18a):

$$\Delta(Z, N+2) = \Delta(Z, N)+F^0_{12}, \tag{27a}$$

$$\Delta(Z+2, N) = \Delta(Z, N)+2(F^0_{13}-F^0_{23}), \tag{27b}$$

$$\begin{aligned}\Delta(Z+2, N+2) &= \Delta(Z, N)+F^0_{12}+2(F^0_{13}-F^0_{23})\\ &= \Delta(Z, N+2)+2(F^0_{13}-F^0_{23}) = \Delta(Z+2, N)+F^0_{12}.\end{aligned} \tag{27c}$$

This means that if we plot the values of $E(j_1)-E(j_2)$ for the above four nuclei as a function of A and join by straight lines the values of the two pairs of isotopes and the values of the two pairs of isotones, we obtain a parallelogram. (In our diagrams only the lines connecting the isotopes are drawn.)

The physical meaning of this parallelogram is as follows: the influence of an additional proton pair in a given shell on the odd-neutron transition from shell j_1 to shell j_2 is independent of the neutron number, and the influence of an additional neutron pair in a given shell is independent of the number of protons. Mathematically this result follows from the linearity of the expressions (18a) (and therefore from the quadratic dependence of the energy (16)) in the neutron and proton numbers, which makes the effects of the neutrons and of the protons on the energy differences independent of each other.

The difference (18a) served only as an example. Evidently, one obtains a parallelogram when considering in the above manner the energy difference of any two configurations $(j_1^{2n_1+1}\, j^{2n_2}\, j^{2p_3}\, j^{2p_4})$ and $(j_1^{2n_1+1\pm k}\, j_2^{2n_2\mp k}\, j_3^{2p_3\pm l}\, j_4^{2p_4\mp l})$, with fixed k and l, in four nuclei (Z, N), $(Z, N+n)$, $(Z+z, N+n)$ and $(Z+z, N)$, with any n and z, provided the z protons entered into the same shells in both the lighter and the heavier isotones, and the n neutrons entered into the same shells in the two elements.

When one encounters such parallelograms in the graphs of the levels systematics, it is but natural to interpret them according to the conditions leading to parallelogram formation set forth in the preceding paragraph. In some cases this suffices to determine uniquely the kind of transition involved in the formation of one state from another and also of the configurations, as will be exemplified in the following sections.

18. Configuration assignment in Kr^{83} and Sr^{85}. A quantitative treatment of the systematics of $E(g_{9/2})-E(p_{1/2})$ in the odd-neutron nuclei. Ground state neutron configurations in some more nuclei

When joining by straight lines the $g_{9/2}$ levels of Kr^{83}, Kr^{85}, Sr^{87} and Sr^{85} (in this order) in fig. 1, one obtains a figure which slightly resembles a parallelogram. This suggests that also in Kr^{83} and Sr^{85} the $p_{1/2}$ state is obtained from the $g_{9/2}$ by a neutron transition from $p_{1/2}$ to $g_{9/2}$, as in the heavier isotopes Kr^{85} and Sr^{87}, and that in passing from Kr^{83} to Sr^{85} the proton pair is added in the same shell, $p_{3/2}$, as in the passage from Kr^{85} to Sr^{87}. This determines the neutron configurations of Kr^{83} and Sr^{85} as $p_{1/2}^2 g_{9/2}^7$ and the proton configuration of Kr^{83} as $p_{3/2}^2 f_{5/2}^6$. Perhaps the small quadrupole moment of Kr^{83} as compared to that of Kr^{85} may be explained by this change from a "single hole" moment to a "three holes" moment, according to (24).

However, the parallelogram is quite distorted, and it is difficult to exclude a priori other possibilities of interpretation. In principle there are four possible configurations for Kr^{83} (the proton configurations are on the left and the neutron configurations on the right):

$$(p_{3/2}^2 f_{5/2}^6;\quad p_{1/2}^2 g_{9/2}^7) \qquad \text{(a)}$$
$$(p_{3/2}^2 f_{5/2}^6;\quad g_{9/2}^9) \qquad \text{(b)}$$
$$(p_{3/2}^4 f_{5/2}^4;\quad p_{1/2}^2 g_{9/2}^7) \qquad \text{(c)}$$
$$(p_{3/2}^4 f_{5/2}^4;\quad g_{9/2}^9), \qquad \text{(d)}$$

and two for Sr^{85}:

$$(p_{3/2}^4 f_{5/2}^6;\quad p_{1/2}^2 g_{9/2}^7) \qquad \text{(a')}$$
$$(p_{3/2}^4 f_{5/2}^6;\quad g_{9/2}^9). \qquad \text{(b')}$$

This gives altogether eight possible interpretations of the levels in the two nuclei Kr^{83}, Sr^{85}, which, too, should be examined with regard to their consistency and their agreement with experiment.

However, all but the parallelogram interpretation (aa′) turn to be inconsistent after a detailed examination. For example, (ab′) cannot stand, as according to it the proton pair added to Kr^{83} entered in Sr^{85} into $p_{3/2}$ which prefers p_n to g_n, and therefore it is impossible that the equilibrium distribution of the neutrons moved in the direction of $g_{9/2}$. (bb′), (ca′) and (da′) are excluded on similar grounds. (ba′) is excluded because according to the parameters obtained by comparing it with the experimental systematics a $p^2_{1/2}g^7_{9/2}$ neutron configuration in Kr^{83} should be more stable than $g^9_{9/2}$, though the latter was assumed to be the ground state configuration. Finally, in the cases of (cb′) and (db′), $\pi_{9/2}-\pi_{1/2}$ and $F_0(p_ng_n)$, respectively, come out negative. It turns out that the similarity in the systematics of the four nuclei, which suggests their interpretation according to the parallelogram principle, originates from a similarity in the structure of the states and the transitions involved in forming them, and is not an accidental resemblance.

It still remains to assign the proton configuration of Zn^{69}. From comparison with the proton configurations of the Ga isotopes (table 6) it is almost certain that it is $p^2_{3/2}$.

When making a comparison between theory and experiment it is convenient to introduce the parameters

$$b = F^0(p_p p_n) - F^0(p_p g_n), \tag{28a}$$

$$a = (a_{1/2} - a_{9/2}) + 2b, \tag{28b}$$

$$f = F^0(p_n g_n), \tag{28c}$$

$$d = F^0(f_p p_n) - F^0(f_p g_n), \tag{28d}$$

$$c = (a_{1/2} - a_{9/2}) + (\pi_{1/2} - \pi_{9/2}) + 3f + 2b + 6d, \tag{28e}$$

$$g = F^0(g_p p_n) - F^0(g_p g_n), \tag{28f}$$

$$e = c + f + 4b + 2g, \tag{28g}$$

and then one sees that in Zn^{69} and Mo^{91} we have the parameters a and e, respectively, to adjust the formulas exactly to the experimental data. The quality of the agreement between theory and experiment will be determined by the calculation for the other nuclei, where we have at our disposal three free parameters to adjust five energy differences.

The results are given in table 5 in the customary form of atomic spectroscopy, where the numbers in the columns headed by the symbols of the parameters are the coefficients of these parameters in the pertinent energy formulas. (For example, the $E(g_{9/2}) - E(p_{1/2})$

of Sr^{85} is given by $c+2b$.) The mean error is 40 keV. For the sake of visualization we plotted in fig. 1 also the theoretical values obtained from the parameters of table 5.

With the aid of the parameters obtained in table 5, the negative sign of d, and (very cautiously) also of table 2, we may determine the neutron configurations in some odd-Z nuclei whose proton

TABLE 5

Comparison of the $E(g_{9/2})-E(p_{1/2})$ Systematics of Odd-N Nuclei with the Theory

Nucleus	Parameters a	c	b	f	e	exp.	cal.	diff.
Zn^{69}	−1					−439	−439	0
Kr^{83}		1				41	61	−20
Sr^{85}		1	2			233	213	20
Kr^{85}		1		1		305	276	29
Sr^{87}		1	2	1		390	428	−38
Zr^{89}		1	4	1		588	579	9
Mo^{91}					1	650	650	0
mean deviation								±21
mean error								±40

Most probable Values of the Parameters

a	439 keV	f	215 keV
b	76 keV	e	650 keV
c	61 keV		

configurations are determined by their nuclear moments. The configurations so determined are marked by c in table 6. An example of the procedure is given in the following section.

19. THE FAILURE OF THE SINGLE-PARTICLE APPROXIMATION TO ACCOUNT FOR THE GROUND STATES OF THE Rb ISOTOPES

Let us use the parameters of table 5 to calculate the difference

$$\begin{aligned} E(p^3_{3/2}f^6_{5/2};\ p^2_{1/2}g^4_{9/2}) - E(p^3_{3/2}f^6_{5/2};\ g^6_{9/2}) = \\ = 2(a_{1/2}-a_{9/2}) + (\pi_{1/2}-\pi_{9/2}) + 4F^0(p_n g_n) + 6\big(F^0(p_p p_n) \\ - F^0(p_p g_n)\big) + 12\big(F^0(f_p p_n) - F^0(f_p g_n)\big) \\ = a+c+f+2b+6d = 6d+867 \text{ keV} \end{aligned} \tag{29}$$

for Rb^{81}, whose proton configuration is $p^3_{3/2}f^6_{5/2}$. From table 5

$$6d = c-a-3f+(\pi_{9/2}-\pi_{1/2}) = (\pi_{9/2}-\pi_{1/2})-1023 \text{ keV},$$

and in order that (29) would be negative, $(\pi_{9/2}-\pi_{1/2})$ should not exceed 156 keV, which certainly cannot be admitted (table 3).

Therefore, (29) is positive and the neutron shell $p_{1/2}$ is already filled in Rb^{81}. The same holds a fortiori for Rb^{83} and Rb^{85}, since here the distribution of the protons favours p_n more than in Rb^{81}, and also in Rb^{87}, where the tendency towards equal distribution of the neutrons between their shells is stronger than in Rb^{81}, owing to their greater number in $g_{9/2}$.

TABLE 6

Ground States Configurations of Odd-Z Nuclei before $N = 50$

Nucleus	Z	N	$I\pm$ Ref.	μ Ref.	Q Ref.	Proton Shells $p_{3/2}$ $f_{5/2}$	Neutron Shells $p_{3/2}$ $f_{5/2}$ $p_{1/2}$ $g_{9/2}$
Cu^{63}	29	34	3/2⁻ 49(d),33	2.226 49	—0.16 49	1 a	? ?
Cu^{65}		36	3/2⁻ 49(d),33	2.386 49	—0.15 49	1 a	? ?
Cu^{67*}		38	3/2⁻ 65			1 a	4 6 a
Ga^{67*}	31	36	3/2⁻ 33			3	? ?
Ga^{69}		38	3/2⁻ 49(d),33	2.017 49	0.243 49	3 a	4 6 a
Ga^{71}		40	3/2⁻ 49(d),33	2.561 49	0.152 49	3 a	4 6 2 c
Ga^{73*}		42	3/2⁻ 33			3	4 6 2 2 c
As^{71*}	33	38	5/2⁻ 30			4 1 c	4 6 2 a
As^{73*}		40	3/2⁻ 61			3 2	4 6 2
As^{75}		42	3/2⁻ 49(d),33	1.439 49	0.3 49	3 2 a	4 6 2 2 c
As^{77*}		44	3/2⁻ 33			3 2	4 6 2 4
Br^{79}	35	44	3/2⁻ 49(d),33	2.106 49	0.33 49	3 4 a	4 6 2 4 c
Br^{81}		46	3/2⁻ 49(d),33	2.269 49	0.28 49	3 4 a	4 6 2 6 c
Br^{83*}		48	3/2⁻ 4			3 4	4 6 2 8
Br^{87*}		52	3/2⁻ 33			3 4	4 6 2 10†
Rb^{81*}	37	44	3/2⁻ 66	2.00 66		3 6 a	4 6 2 4 c
Rb^{83*}		46	5/2⁻ 67			4 5 a	4 6 2 6 c
Rb^{85}		48	5/2⁻ 49(d),33	1.353 49	0.31 68	4 5 a	4 6 2 8 c
Rb^{87*}		50	3/2⁻ 49(d),33	2.749 49	0.15 68	3 6 a	4 6 2 10 a

† There is an additional neutron pair in $d_{5/2}$ or in $g_{7/2}$.

This excludes the paparently simple explanation of the change of the ground state spin from 3/2 in Rb^{81} to 5/2 in $Rb^{83, 85}$ and again to 3/2 in Rb^{87} (table 6) by the assumption that in Rb^{83} the last neutron pair entered a shell preferring p_p to f_p, while in Rb^{87} the last neutron pair entered a shell preferring f_p to p_p. The above considerations show that from Rb^{81} onwards all the neutrons enter the same $g_{9/2}$ shell.

Therefore we have to assume that at least one of the above mentioned spin-changes is due to configuration-interaction, and the lower unperturbed ground J-state is not the one observed

experimentally. With the parameters of table 5 we may examine such explanations.

As the proton shells are filled but for one hole, only the neutron shells can cause an interaction, and the non-diagonal element depends only on the neutron interaction and is the same between the two $p_{3/2}$ and between the two $f_{5/2}$ interacting states (section 8). The distance between the diagonal elements is

$$E(p_{3/2}^3 f_{5/2}^6;\ p_{1/2}^2 g_{9/2}^{2n-2}) - E(p_{3/2}^3 f_{5/2}^6; g_{9/2}^{2n}) = 2(a_{1/2}-a_{9/2}) + (\pi_{1/2}-\pi_{9/2}) + (2n-2)f + 6b + 12d \quad (30a)$$

between the two $p_{3/2}$ states, and

$$E(p_{3/2}^4 f_{5/2}^5;\ p_{1/2}^2 g_{9/2}^{2n-2}) - E(p_{3/2}^4 f_{5/2}^5;\ g_{9/2}^{2n}) = 2(a_{1/2}-a_{9/2}) + (\pi_{1/2}-\pi_{9/2}) + (2n-2)f + 8b + 10d \quad (30b)$$

between the two $f_{5/2}$ states. (30b) is greater than (30a) by the positive quantity $2b-2d$ and therefore the $p_{3/2}$ are more strongly repelled by the interaction, and the only possibility is that in Rb^{81} the unperturbed ground state had to be $f_{5/2}$, but because of the interaction $p_{3/2}$ was depressed below it. In Rb^{83} and Rb^{85} the interaction did not suffice and the $f_{5/2}$ remained lower, whereas in Rb^{87} the additional $g_{9/2}^2$ neutron pair pulled the protons in the direction of $f_{5/2}$. In fact, it is but plausible that the interaction will be most pronounced in Rb^{81}, where the number of neutrons is the smallest, and therefore the two unperturbed states are closest in their energy.

However, the explanation fails quantitatively. Experimentally the differences $E(f_{5/2})-E(p_{3/2})$ are known [32, 33] in Rb [85, 87], and one has (in a self-evident notation)

$$E_{5/2}(Rb^{85}) - E_{3/2}(Rb^{85}) = 149 \text{ keV} \quad (31a)$$

$$E_{5/2}(Rb^{87}) - E_{3/2}(Rb^{87}) = -403 \text{ keV}. \quad (31b)$$

From the linearity of $E_{5/2}-E_{3/2}$ in the neutron number one obtains

$$E_{5/2}(Rb^{81}) - E_{3/2}(Rb^{81}) = 1253 \text{ keV}. \quad (31c)$$

By assuming linearity we neglected configuration-interaction, but the consideration of the interaction, which exists only in Rb^{85}, may only increase (31c), according to the concludion obtained above from (30a, b). Therefore, if the above explanation is to hold, the interaction would have to depress the $p_{3/2}$ state of Rb^{81} by at least amount.

But the non-diagonal element between the two $p_{3/2}$ states is given according to (15a) and to table 5 by

$$(p_{1/2}^2 g_{9/2}^4 | \Sigma V_{ik} | g_{9/2}^6) = -3F^0(p_n g_n) = -645 \text{ keV} \qquad (32)$$

and cannot compensate a difference of 1253 keV.

As a matter of fact, it could hardly be expected to get a sufficient depression in our approximation, as 1253 keV is very much above the first excited state of 620 keV of the "core" Kr^{80} [34]), and there are undoubtedly other states in this region, in which the core is not in a $(J = 0)$-state, and which may also interact with the $p_{3/2}$ single-particle state and depress it. It seems that there is a strong interaction between the low lying states of Rb^{81}, which makes the single-particle approximation invalid.

A similar conclusion was arrived at by Thulin [32]) when investigating the level scheme of Br^{79}, and by Schardt and Welker [35]) when investigating As^{75}. Additional experimental data, both in odd-Z and in odd-N nuclei, are needed to enable one a quantitative treatment which will elucidate the structure of these states.

CHAPTER 4. THE COMPETITION BETWEEN THE PROTON SHELLS $2p_{1/2}$, $1g_{9/2}$ AND THE COMPETITION BETWEEN THE NEUTRON SHELLS $2d_{5/2}$, $1g_{7/2}$

20. A SURVEY OF THE EXPERIMENTAL DATA

The discussion in this chapter will not be quantitative, as a glance at the systematics of $E(g_{9/2}) - E(p_{1/2})$ in the odd-Z nuclei

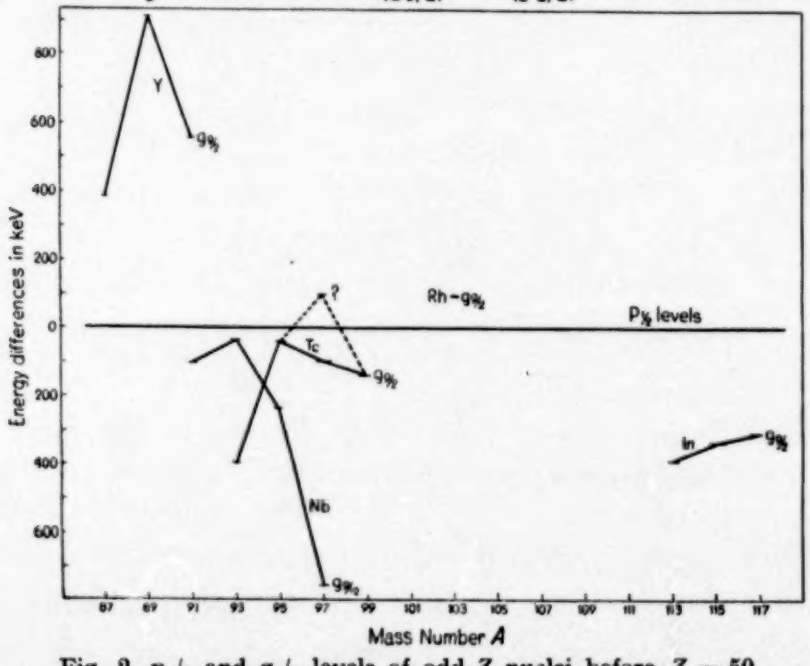

Fig. 2. $p_{1/2}$ and $g_{9/2}$ levels of odd-Z nuclei before $Z = 50$.
References: $Y^{87, 89, 91}$ (33); Nb^{91} (69); $Nb^{93, 95, 97}$ (33); Tc^{93} (71); $Tc^{95, 99}$ (33); Tc^{97} (4); Rh^{103} (83); $In^{113, 115}$ (33); In^{117} (74).

(fig. 2) suffices to show that it is not regulated by the linear formulas of chapter 2, or that there is a strong configuration-interaction, if one wants to use the language of the shell model.

The discussion will also be very fragmentary. In all the odd-N nuclei pertaining here, not a single quadrupole moment is known, and therefore no direct information is available on the filling of the $d_{5/2}$ and $g_{7/2}$ shells. Also only very few J-assigned excited states are known for odd N [33, 36], which deprives us also of the second kind of information which may by utilized in determining the filling order.

TABLE 7

Ground State Configurations of Odd-Z Nuclei before $Z = 50$

Nucleus	Z	N	$I\pm$	Ref.	μ	Ref.	Q	Ref.	Proton Shells $p_{1/2}$	$g_{9/2}$	Neutron Shells $g_{9/2}$	$d_{5/2}$	$g_{7/2}$	$h_{11/2}$	
Y^{87*}	39	48	1/2⁻	33					1	a	8				c
Y^{89}		50	1/2⁻	49(d),33	—0.137	49			1	a	10				a
Y^{91*}		52	1/2⁻	33					1	a	10	?	?		
Nb^{91*}	41	50	9/2†	69					2	1 c	10				a
Nb^{93}		52	9/2⁺	49(d),33	6.167	49	—0.2	70	?	?	10	?	?		
Nb^{95*}		54	9/2⁺	33					?	?	10	?	?		
Nb^{97*}		56	9/2⁺	33					?	?	10	?	?		
Tc^{93*}	43	50	9/2⁺	71					2	3 c	10				a
Tc^{95*}		52	9/2⁺	33					?	?	10	?	?		
Tc^{97*}		54	9/2⁺?	4					?	?	10	?	?		
Tc^{99*}		56	9/2⁺	49(d),33	5.681	49	0.34††	49	?	?	10	?	?		
Tc^{101*}		58	9/2⁺	36					?	?	10	?	?		
Rh^{103}	45	58	1/2⁻	49(d),33	—0.088	73			1	6 a	10	?	?		
Rh^{105*}		60	7/2⁺†††	33					?	?	10	?	?		
Ag^{107}	47	60	1/2⁻	49(d),33	—0.113	49			1	8 a	10	?	?		
Ag^{109}		62	1/2⁻	49(d),33	—0.130	49			1	8 a	10	?	?		
Ag^{111*}		64	1/2⁻	49(d),33					1	8 a	10	6	8		a
Ag^{113*}		66	1/2⁻	4					1	8 a	10	6	8	2	
Ag^{115*}		68	1/2⁻	4					1	8 a	10	6	8	4	
In^{111*}	49	62	9/2⁺	33					2	9 a	10	?	?		
In^{113}		64	9/2⁺	49(d),33	5.522	49	1.181	49	2	9 a	10	6	8		a
In^{115*}		66	9/2⁺	49(d),33	5.534	49	1.198	49	2	9 a	10	6	8	2	b?
In^{117*}		68	9/2⁺	74					2	9 a	10	6	8	4	b?

† The announcement of Hayward and the ground state spin 9/2⁺ which it suggests were pointed out to me by Dr. M. Goldhaber.

†† The authors [72]) give the value 0.34 ± 0.34, or even a small negative value.

††† Not a single particle state.

The only approximately linear behaviour is seen in the $E(g_{9/2})-E(p_{1/2})$ separation of the three In isotopes, $In^{113, 115, 117}$. However, in them the neutron $d_{5/2}$ and $g_{7/2}$ shells are already full, and this linearity cannot answer anything concerning their filling. Anticipating the results of the following chapter that the first pairs of neutrons, after the $d_{5/2}$ and $g_{7/2}$ shells are closed, enter shell $h_{11/2}$, we explain the approximate linearity by the addition of $h_{11/2}$ neutrons from one isotope to its heavier neighbour. The decrease in the excitation energy of the $p_{1/2}$-state shows then that h_n prefers g_p to p_p (the proton configurations in the various J-states are uniquely determined in In (table 7)), which is in accordance with table 2.

21. Ground state configurations of some odd-Z nuclei and consistency considerations

In section 16 we saw that in Zr^{90}, with 50 neutrons, the last proton pair entered the $p_{1/2}$ shell. This enables us to assert that the $p_{1/2}$ shell is filled also in all the odd-proton nuclei with a ground state $g_{9/2}$ and 50 neutrons, like Nb^{91} and Tc^{93}: In fact we have from Zr^{90} that $2(A_{1/2}-A_{9/2})+(\Pi_{1/2}-\Pi_{9/2})$ is positive, which implies that

$$E(p_{1/2}^2 g_{9/2}^p)-E(g_{9/2}^{p+2}) = 2(A_{1/2}-A_{9/2})+(\Pi_{1/2}-\Pi_{9/2})+p\,.\,F^0(p_p g_p) \quad (33)$$

is positive too.

The difference $E(g_{9/2})-E(p_{1/2})$ is given by

$$E(p_{1/2}^2 g_{9/2}^p)-E(p_{1/2} g_{9/2}^{p+1}) = (A_{1/2}-A_{9/2})+(\Pi_{1/2}-\Pi_{9/2})+\tfrac{1}{2}(p-1)F^0(p_p g_p), \quad (34)$$

and therefore it should increase by $F^0(p_p g_p)$ from Nb^{91} to Tc^{91}. Experimentally, it increases by 285 keV, which is rather close to the approximate value of 240 keV for $F^0(pg)$ according to table 2, or to the value of 215 keV obtained for $F^0(p_n g_n)$ from table 5. However, the values of these experimental differences themselves are found to be too high (at least for Nb^{91}) when we insert in the last formula the values obtained for its terms from the experimental data of Y^{89} and from table 2. It is probable, therefore, that an interaction exists, which depresses the $g_{9/2}$ states much below their unperturbed values. It is quite probable that without this interaction the ground state of Nb^{91} would really be $p_{1/2}$.

Goldhaber and Hill [4]) suggested that Tc^{97} has a ground state $g_{9/2}$, as they thought that the maximum of the $g_{9/2}$ curve should occur for $N = 50$. The experimental discovery of the M4 transitions of Nb^{91} and Tc^{93} invalidated this maximum-assumption (fig. 2). Therefore, it would be interesting to find experimentally whether the ground state of Tc^{97} is $g_{9/2}$ or $p_{1/2}$. If the latter would be found to be the case we would have a much closer resemblance of the graphs of the three elements Y, Nb and Tc to each other than in figure 75 of Goldhaber and Hill, and also another regularity replacing that of the $N = 50$ extremum, according to which the N of the extremum is shifted by two with the addition of each pair of protons, being 50 for Y^{89}, 52 for Nb^{93} and 54 for Tc^{97}.

CHAPTER 5. THE COMPETITION BETWEEN THE NEUTRON SHELLS $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$ AND THE COMPETITION BETWEEN THE PROTON SHELLS $2d_{5/2}$, $1g_{7/2}$

22. A QUALITATIVE DISCUSSION OF THE SYSTEMATICS OF THE EVEN ISOTOPES OF Sn, Te AND Xe

From the ground state spins of the odd-N nuclei of table 8 it is evident that many neutron pairs enter $h_{11/2}$ before $s_{1/2}$ and $d_{3/2}$ are filled. The negative quadrupole moment of Xe^{131} shows that it has at least ten $h_{11/2}$ neutrons while $d_{3/2}$ is devoid of neutron pairs, and therefore it is but plausible that for smaller neutron numbers all neutron pairs went into $h_{11/2}$.

The systematics of the first excited 2^+-state in the even isotopes of Sn, Te and Xe is in agreement with this assumption (fig. 3 †). Generally speaking, the difference $E(0^+)-E(2^+)$ is very high in Sn, much lower in Te, and still somewhat lower in Xe, but it does not change much from one isotope to another of the same element (if we disregard the heavier Xe isotopes). Assuming pure states this may mean that in Sn, where the proton number is magic, the pair which may be excited to a $j^2(2)$-state must be in a neutron shell whose excitation energy is very high, and according to table 3 this shell is $h_{11/2}$. From the fact that from Sn^{116} to Sn^{122} one does not observe †† a lower 2^+ state corresponding to a pair $d^2_{3/2}(2)$, one may infer that also in Sn^{122}, with eight neutrons outside the

† We assume that the first excited state of Sn^{116} is 2^+ too.

†† We assume that such a state will not appear also in Sn^{118}. However, it is necessary to check this experimentally.

closed shells $d_{5/2}$ and $g_{7/2}$, the $d_{3/2}$ shell is still empty. However, the new discovery of the 2^+ state of 1300 keV in Sn^{114} [37]), which consists of closed shells according to the above ideas, might invalidate our interpretation of the Sn—2^+-states, if it would turn

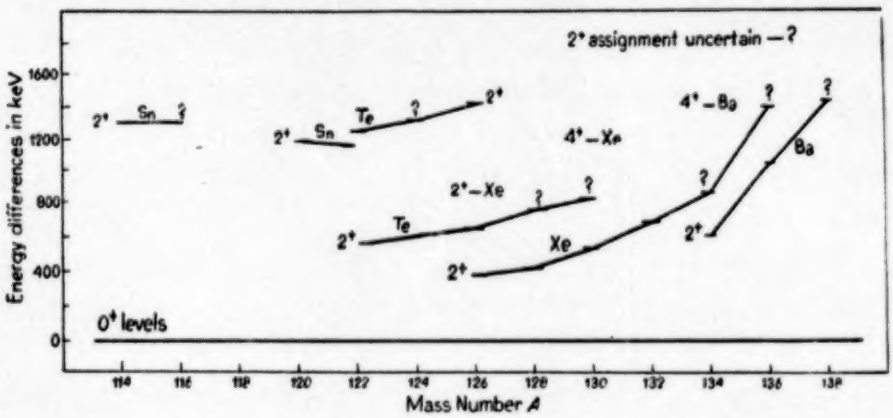

Fig. 3—0^+, 2^+ and 4^+ levels of the even-even isotopes of Sn, Te, Xe and Ba. References: Sn^{114} (37); Sn^{116} (33); Sn^{120} (84); Sn^{122} (85); $Te^{122, 124}$ (33); Te^{126} (86); Te^{128} (87); Te^{130} (88); Xe^{126} (86); Xe^{128} (89); Xe^{130} (90); Xe^{132} (91); $Xe^{134, 136}$ (92); Ba^{134} (93); Ba^{136} (94); Ba^{138} (89).

out that there are no lower excited states; since, if its shells are really closed, its first excited state should be 0^+, whereas, if $h_{11/2}$ begins to be filled before $d_{5/2}$ and $g_{7/2}$ are closed, there should be a lower 2^+ excited state.

If the above interpretation of the Sn—2^+ states is correct, one may expect these 2^+ states to appear in the heavier Cd isotopes, too. However, the second excited state of 1278 keV of Cd^{114} was recently [38, 39, 37]) found to be 4^+. The first excited 2^+ states of $Cd^{114, 116}$ should probably be attributed to the two proton holes which are absent in Sn, and the approximate constancy of the 2^+ excitation energy in $Cd^{110, 112, 114, 116}$ [40, 41]) suggests that this applies also to the lighter Cd isotopes. This shows that the proton configuration of these Cd nuclei is $p^2_{1/2}\, g^8_{9/2}$ and not $g^{10}_{9/2}$, which is reasonable considering the strong tendency of such a high proton number towards distribution between the two shells. We met the same situation in the corresponding neutron shells of Rb^{85}. However, it is perhaps too early to say anything in this connection before considering quantitatively the systematics of the even isotopes of Mo, Ru, Pd and Cd.

In the Te nuclei there is a 2^+ state of about the same excitation energy as in the Sn nuclei, which suggests that this too is formed by a $h^2_{11/2}(2)$ pair. But there is also another 2^+ state which is much

lower, and may plausibly be attributed to the opening of the proton pair which was not present in Sn. As in $Sb^{123, 125}$ a single proton goes to $g_{7/2}$ (table 11) and the pairing energy is also higher for $g_{7/2}$ than for $d_{5/2}$ (table 3), it is plausible that the proton pair in their isotones $Te^{124, 126}$ also goes to $g_{7/2}$. The approximate constancy of the excitation energy in $Te^{122, 124, 126}$ suggests that in Te^{122}, too, the protons are in the $g_{7/2}$ shell.

The fact that in Te^{126} there is no second 2^+ level below the $h^2_{11/2}(2)$ level suggests that the neutron shell $d_{3/2}$ is empty also for ten neutrons outside the closed shells $d_{5/2}$ and $g_{7/2}$. Concerning the filling of $s_{1/2}$ we cannot say anything by a similar argument, as it cannot produce a $(J = 2)$-state. The question may be answered by finding experimentally the first excited states of $Sn^{124, 126}$ and the second excited state of Te^{128}, since if the sixth neutron pair also went into $h_{11/2}$, the graphs of these states cannot continue up to Sn^{126} and Te^{128}, whereas if the graphs will be found to continue smoothly, the sixth neutron pair should have entered $s_{1/2}$. The question of when $s_{1/2}$ first becomes filled is very important for the exact treatment of the entire systematics of the odd-N nuclei, which cannot be carried through by assuming pure configurations, as will be seen in section 25.

Looking at the $Xe-2^+$ levels we see in Xe^{126} clearly how the first 2^+ state of Te is split into two $Xe-2^+$ states, which shows that the proton configurations of the 2^+ states in the Xe nuclei should be very mixed. The same conclusion arises also from the steep rise of the $Xe-2^+$ line towards Xe^{136} with 82 neutrons, and applies, therefore, to the $Ba-2^+$ states, too. Finally, the high value of about 2 of the ratio $(E(0^+)-E(4^+))/(E(0^+)-E(2^+))$ in Xe^{130} and Ba^{134}, where according to table 3 it should be about 1.2, might indicate a strong mixing of the states in these nuclei. However, here the deviation from table 3 may also be due to the use of delta-forces rather than to mixing.

Nevertheless, we shall see in the following sections that in a certain sense it may be possible to assign to $g_{7/2}$ the protons of Xe and Ba in their ground states.

23. The competition between the neutron shells $3s_{1/2}$ and $2d_{3/2}$ in the odd-neutron isotopes of Sn, Te, Xe and Ba

In the preceding section we saw that it is plausible that up to Sn^{122} and Te^{126} no neutron pairs entered $d_{3/2}$. Therefore, we assume

this also for the neutron configurations in the $s_{1/2}$ ground states of $Sn^{117, 119}$ and $Te^{121, 123, 125}$ (table 8).

TABLE 8

Ground State Configurations of Odd-N Nuclei before $N = 82$

Nucleus	Z	N	$I\pm$	Ref.	μ	Ref.	Q	Ref.	Proton Shells $g_{9/2}$	$d_{5/2}$	$g_{7/2}$	Neutron Shells $s_{1/2}$	$d_{3/2}$	$h_{11/2}$
Cd^{111}	48	63	$1/2^+$	49(d),33	−0.592	49			8		e	1†		
Cd^{113}	48	65	$1/2^+$	49(d),33	−0.620	49			8		e	1		a
Sn^{115}	50		$1/2^+$	49(d),33	−0.913	49			10		a	1		a
Cd^{115*}	48	67	$1/2^+$	33					8		e	1	2	
Sn^{117}	50		$1/2^+$	49(d),33	−0.995	49			10		a	1	2	b, e
Sn^{119}	50	69	$1/2^+$	49(d),33	−1.041	49			10		a	1	4	b, e
Te^{121*}	52		$1/2^+$	33					10	2	b	1	4	b
Sn^{121*}	50	71	$3/2^+$	33					10		a	?	?	?
Te^{123}	52		$1/2^+$	49(d),33	−0.732	49			10	2	b	1	6	b
Sn^{123*}	50	73	$3/2^+$	4					10		a	?	?	?
Te^{125}	52		$1/2^+$	49(d),33	−0.882	49			10	2	b	1	8	b
Xe^{127*}	54		$3/2^+$	42					10	? ?		?	?	?
Sn^{125*}	50	75	$11/2^-$	?4					10		a	?	?	?
Te^{127*}	52		$3/2^+$	33					10	? ?		?	?	?
Xe^{129}	54		$1/2^+$	49(d),33	−0.773	49			10	4	b	1	10	b
Te^{129*}	52	77	$3/2^+$	33					10	? ?		?	?	?
Xe^{131}	54		$3/2^+$	49(d),33	0.687	49	−0.12	49	10	4	b		1 12	b
Ba^{133*}	56		$1/2^+$	33					10	6	b	1	12	b
Te^{131*}	52	79	$3/2^+$	33					10	2	b	2	1 12	b
Xe^{133*}	54		$3/2^+$	33					10	4	b	2	1 12	b
Ba^{135}	56		$3/2^+$	49(d),33	0.835	49			10	6	b	2	1 12	b
Te^{133*}	52	81	$3/2^+$	33					10	2	b	2	3 12	a
Xe^{135*}	54		$3/2^+$	33					10	4	b	2	3 12	a
Ba^{137}	56		$3/2^+$	49(d),33	0.935	49			10	6	b	2	3 12	a

† In Cd^{111} there are two holes either in the $d_{5/2}$ or in the $g_{7/2}$ neutron shell.

The simplest assumption about the excited $d_{3/2}$ states in these nuclei is that they are formed by the transition of the odd neutron from $s_{1/2}$ to $d_{3/2}$. Looking then at the graph of the difference $E(s_{1/2})-E(d_{3/2})$ in fig. 4 we see that it is more or less linear in the neutron numbers and in the proton numbers of Sn, Te, Xe and Ba. This suggests that also the protons added from element to

element enter the same shell. From the discussion of the odd-Z nuclei (section 26) this shell can then be only $g_{7/2}$.

With these assumptions the interpretation of the systematics of $E(s_{1/2})-E(d_{3/2})$ is very simple: the neutrons added in $h_{11/2}$ in the

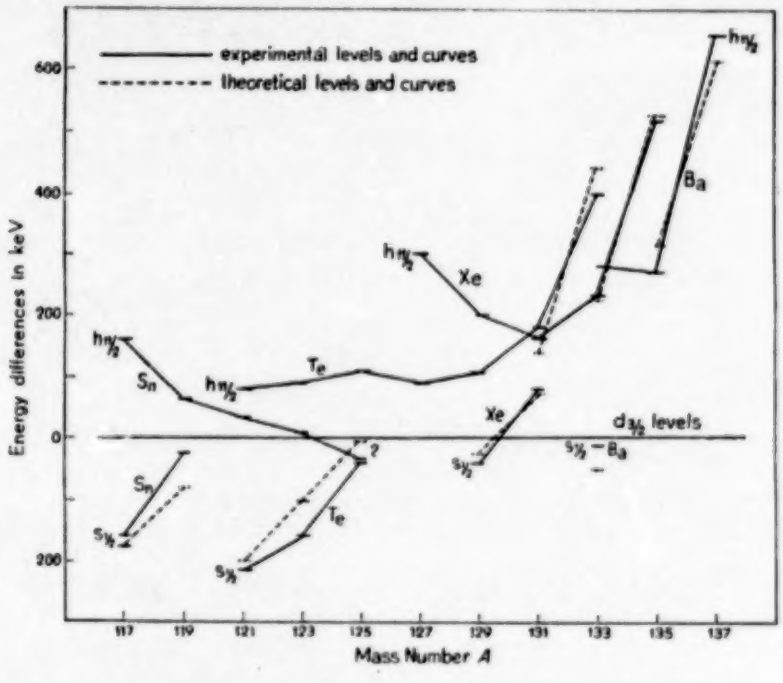

Fig. 4. $s_{1/2}$, $d_{3/2}$ and $h_{11/2}$ levels of the Sn, Te, Xe and Ba odd-mass nuclei. References: $Sn^{117,\,119}$ (33); $Sn^{121,\,123,\,125}$ (4); $Te^{121,\,123,\,125,\,127,\,129,\,131,\,133}$ (33); Xe^{127} (42); $Xe^{129,\,131,\,133,\,135}$ (33); $Ba^{133,\,135,\,137}$ (33).

heavier isotopes of each element prefer $d_{3/2}$ to $s_{1/2}$ (which preference is in accordance with table 2), and therefore it is always easier to excite the $d_{3/2}$ state when neutrons are added. In Xe^{131} the influence of the $h_{11/2}$ neutrons is already so strong that in the ground state the odd neutron is in $d_{3/2}$.

On the other hand, for a fixed neutron number it is always more difficult to excite the $d_{3/2}$ state when the number of protons is larger. This seems to show that g_p prefers s_n to d_n, which is in contradiction with table 2. However, according to this table the difference $F^0(\mathrm{gd})-F^0(\mathrm{gs})$ is rather small even for particles of the same kind moving in the same central field.

When examining the quantitative agreement between the interpretation proposed above and the experimental systematics, it is convenient to introduce the parameters (such convenient combinations are defined anew in each chapter)

$$a = a_{1/2} - a_{3/2}, \tag{35a}$$

$$b = F^0(h_n s_n) - F^0(h_n d_n), \tag{35b}$$

$$c = F^0(g_p s_n) - F^0(g_p d_n). \tag{35c}$$

The comparison of the theory with the experimental data is given in table 9. The negative value of b shows the preference of d_n

TABLE 9

Comparison of the $E(s_{1/2})-E(d_{3/2})$ Systematics of Odd-N Nuclei with the Theory

Nucleus	Parameters a	b	c	exp.	cal.	diff.
Sn^{117}	1	1		162	178	−16
Sn^{119}	1	2		24	78	−54
Te^{121}	1	2	2	213	202	11
Te^{123}	1	3	2	159	102	57
Te^{125}	1	4	2	35	2	33
Xe^{129}	1	5	4	39	26	13
Xe^{131}	1	6	4	−80	−74	−6
Ba^{133}	1	6	6	12	50	−38
mean deviation						±34
mean error						±43

Most Probable Values of the Parameters

a	278 keV	c	62 keV
b	−100 keV		

to s_n by h_n, and the positive value of c shows the preference of s_n to d_n by g_p. The mean error is 43 keV. To see the roughness of the approximation we plotted in fig. 4 also the theoretical lines obtained from the parameters of table 9.

An extrapolation of the $s_{1/2}$ lines of fig. 4 to the heavier Sn and Te isotopes, which is justified if all their neutron pairs are also in $h_{11/2}$, might explain why their ground states are already $d_{3/2}$. However, the fact that in these nuclei the ground state is $d_{3/2}$ does not necessarily mean that all their neutron pairs are $h_{11/2}$, because it is possible that at the same time the $s_{1/2}$ shell is filled. The question could be answered experimentally by searching for the $s_{1/2}$ levels of these isotopes. If the lines of the $s_{1/2}$ levels would be found to continue approximately linearly beyond Sn^{119} and Te^{125}, it is probable that the first alternative holds, whereas, if they would be found to be brusquely interrupted, this would probably be due to the completion of the $s_{1/2}$ shell and the corresponding

change in the transition leading to the formation of the $s_{1/2}$ states.

On the other hand, we did not find it possible to understand by our pure-state approximation why the ground state of Xe^{127} should be $d_{3/2}$ as claimed by Mathur and Hyde [42]). We suggested ground state configurations $g^{p}_{7/2} s_{1/2} h^{n-1}_{11/2}$ both for Te^{121} with $(p, n) = (2, 5)$ and for Ba^{133} with $(p, n) = (6, 13)$. Owing to the linearity of all the energy differences in our approximation in the neutron and in the proton numbers this should then be the most stable configuration also for Xe^{127} with $(p, n) = (4, 9)$. It is very desirable to find in a more direct way the spin of this nucleus.

24. The systematics of the $h_{11/2}$ states when $h_{11/2}$ is already filled in the ground state

Looking at fig. 4 and searching for a behaviour reminding equations (18), we find some resemblance to them only in the graphs of the $h_{11/2}$ levels in the last two isotopes of each of Te, Xe and Ba. In fact, one sees two parallelograms (rather distorted, it should be admitted), obtained by joining by straight lines the $h_{11/2}$ levels of Te^{131}, Te^{133}, Xe^{135} and Xe^{133} (in this order), and the $h_{11/2}$ levels of Xe^{133}, Xe^{135}, Ba^{137} and Ba^{135}, respectively. The resemblance to the linear formulas is much stronger here than in the $s_{1/2}$ graphs of the previous section, which suggests that the linear pure-configuration approximation will be fairly good.

In Te^{133}, Xe^{135} and Ba^{137} the $h_{11/2}$ state may be produced from the ground state $d_{3/2}$ only by a neutron transition from the $h_{11/2}$ to the $d_{3/2}$ shell (table 8). The increase in the excitation energy of the $h_{11/2}$ state is the same in the passage from Te^{133} to Xe^{135} and in the passage from this to Ba^{137}, which shows that the protons are added twice in the same shell. The proton configurations $d^{2}_{5/2} g^{4}_{7/2}$ or $d^{p}_{5/2} g^{2}_{7/2}$ are excluded by consistency considerations, because if $d^{2}_{5/2}$ is more stable than $g^{2}_{7/2}$, $d^{4}_{5/2} g^{2}_{7/2}$ is also more stable than $d^{2}_{5/2} g^{4}_{7/2}$ according to the odd-mass analogue of (16). Therefore all the protons should be in the same shell, and from section 26 this shell is $g_{7/2}$. The increase in the excitation energy of the $h_{11/2}$ states from Te to Xe to Ba shows that g_p prefers h_n to d_n.

According to the parallelogram principle (section 17), Te^{131}, Xe^{133} and Ba^{135} should also have equal neutron configurations, their $h_{11/2}$ state is also obtained from the $d_{3/2}$ by a neutron transition from the $h_{11/2}$ to the $d_{3/2}$ shell, and their proton configurations are the same as in the heavier isotopes. There are three possibilities for the

neutron configuration of the three nuclei: $s^2_{1/2}d_{3/2}h^{12}_{11/2}$, $d^3_{3/2}h^{12}_{11/2}$ and $s^2_{1/2}d^3_{3/2}h^{10}_{11/2}$, the last of which is excluded by the increase of the $h_{11/2}$ excitation energy from the lighter to the heavier isotopes. Which of the first two is the correct one, may most directly be decided by a measurement of the sign of the quadrupole moment of either of the three nuclei. Meanwhile, however, there is strong support in favour of the first alternative. In the first place, if the proton configuration is the same in Ba^{133} as in Ba^{135}, $g^6_{7/2}$, consistency requires that Ba^{135} will have an $s^2_{1/2}d_{3/2}h^{12}_{11/2}$ ground state configuration, since $s_{1/2}d^2_{3/2}h^{12}_{11/2}$ would anyhow be more stable than $d^3_{3/2}h^{12}_{11/2}$, as indicated by the ground state configuration $s_{1/2}h^{12}_{11/2}$ of Ba^{133} (table 8). Also quantitatively, if the alternative $d^3_{3/2}h^{12}_{11/2}$ held, the large increase in the excitation energy of the $h_{11/2}$ state from Te^{131}, Xe^{133}, Ba^{135} to Te^{133}, Xe^{135}, Ba^{137} would equal $F^0(s_n h_n) - F^0(s_n d_n)$, which is expected to be negative from table 2, whereas if the configuration is $s^2_{1/2}d_{3/2}h^{12}_{11/2}$, this increase equals $F^0(h_n d_n)$, which is much more reasonable according to table 2, and also to the order of magnitude of the parameters obtained from the experimental systematics in tables 5 and 9.

An $s^2_{1/2}d_{3/2}h^{12}_{11/2}$ configuration rather than $s_{1/2}d^2_{3/2}h^{12}_{11/2}$ in Te^{131} and Xe^{133} means that after $h_{11/2}$ is filled, the next neutron pair enters $s_{1/2}$ and not $d_{3/2}$, though the single-particle binding energy is larger in $d_{3/2}$, as indicated by the $d_{3/2}h^{12}_{11/2}$ configurations of Te^{129}(?)

TABLE 10

Comparison of the $E(d_{3/2})-E(h_{11/2})$ Systematics of $Te^{131, 133}$, $Xe^{133, 135}$ and $Ba^{135, 137}$ with the Theory

Nucleus	Parameters d	e	f	exp.	cal.	diff.
Te^{131}	1			177	138	39
Xe^{133}	1	2		233	226	7
Ba^{135}	1	4		269	315	−46
Te^{133}	1		1	400	439	−39
Xe^{135}	1	2	1	520	527	− 7
Ba^{137}	1	4	1	661	615	46
mean deviation						±35
mean error						±49

Most Probable Values of the Parameters

d	138 keV	f	301 keV
e	44 keV		

and of Xe^{131}. This may happen only if the pairing energy is higher for $s_{1/2}$ than for $d_{3/2}$, which is confirmed beyond any doubt by table 3.

To see the magnitude of the disagreement with the theory caused by the distortion of the parallelograms, we performed a least-square calculation also for the present six energy differences, whose results are given in table 10. We used the parameters

$$e = F^0(g_p h_n) - F_0(g_p d_n), \tag{36a}$$

$$f = F^0(h_n d_n), \tag{36b}$$

$$d = (a_{11/2} - a_{3/2}) + (\pi_{11/2} - \pi_{3/2}) + (F^0(s_n h_n) - F^0(s_n d_n)) + 2e - 5f. \tag{36c}$$

In fig. 4 we plotted in broken lines the theoretical graph corresponding to the parameters of table 10.

25. The systematics of the $h_{11/2}$ states when $h_{11/2}$ is being filled with neutron pairs, and the invalidity of the pure-configuration approximation in the middle of the shell

Looking at the $h_{11/2}$ lines of the lighter isotopes in fig. 4 we do not see any resemblance to the linear formulas (18). However, if we draw the graphs of $E(s_{1/2}) - E(h_{11/2})$ (fig. 5), we find some

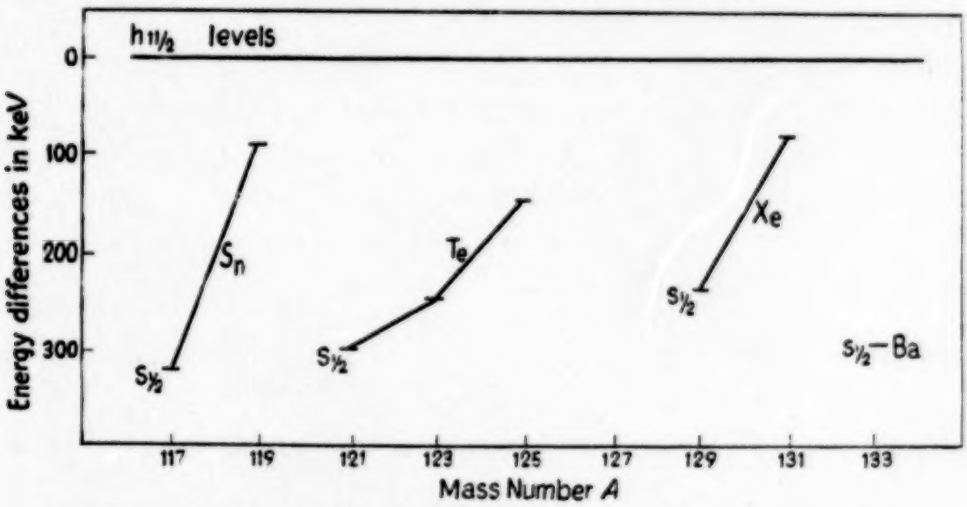

Fig. 5. $E(s_{1/2}) - E(h_{11/2})$ of the Sn, Te, Xe and Ba odd-mass nuclei. References: see fig. 4.

resemblance to the behaviour of the $E(s_{1/2}) - E(d_{3/2})$ in fig. 4. The excitation energy of the $h_{11/2}$ state decreases as neutrons are added in the same element, and for a fixed neutron number this excitation energy increases when protons are added from element

to element. To a certain approximation, which is even worse than that of section 23, we have again parallel equidistant lines, and a qualitative explanation seems possible.

The $h_{11/2}$ states may be obtained from the ground states either by a neutron transition from the $s_{1/2}$ or the $d_{3/2}$ shell to the $h_{11/2}$ shell or by a transition in the opposite direction. In the first case the excitation energy of the $h_{11/2}$ states would have to increase in the heavier isotopes. Thus the experimental facts exclude this possibility.

In the second alternative the pair formed by the ground-state odd neutron and the neutron that left $h_{11/2}$ may remain in the $s_{1/2}$ or in the $d_{3/2}$ shell. In the last case, when a neutron pair is added, the difference $E(d_{3/2})-E(h_{11/2})$ (fig. 4) should decrease by the same amount it increases from Te^{131}, Xe^{133}, Ba^{135} to Te^{133}, Xe^{135}, Ba^{137}, respectively, contrary to the facts.

Finally, the remaining alternative, that in the $h_{11/2}$ state a neutron pair is in the $s_{1/2}$ shell, though seeming at first sight to account for the systematics satisfactorily, is also found to be untenable, both if we assume that the $s_{1/2}$ shell is still empty in the ground state of the heavier isotopes of Sn and Te and all their neutron pairs are in the $h_{11/2}$ shell, or if we assume that the $s_{1/2}$ shell is already full in these heavier isotopes. We are now going to demonstrate this untenability.

The assumption that $s_{1/2}$ is filled in the $h_{11/2}$ states in the lighter isotopes is inconsistent with the assumption that in the $d_{3/2}$ ground states there are no neutrons in $s_{1/2}$ even in the heavier isotopes, where there should be a stronger tendency to distribute the neutrons in the different shells. The assumption for the $h_{11/2}$ state of Sn^{117} leads, for example, to

$$E(s_{1/2}^2 h_{11/2})-E(h_{11/2}^3) = 2(a_{1/2}-a_{11/2})+(\pi_{1/2}-\pi_{11/2})+F^0(s_n h_n) > 0, \quad (37a)$$

while the suggestion that the neutron configuration in the ground state of Sn^{121} is $d_{3/2}h_{11/2}^6$ leads to

$$E(s_{1/2}^2 d_{3/2} h_{11/2}^4)-E(d_{3/2} h_{11/2}^6) = 2(a_{1/2}-a_{11/2})+(\pi_{1/2}-\pi_{11/2}) + (F^0(s_n d_n)-F^0(d_n h_n))+4F^0(s_n h_n) < 0, \quad (37b)$$

and from these two one obtains

$$F^0(d_n h_n)-F^0(s_n d_n)-3F^0(s_n h_n) > 0. \quad (37c)$$

One observes now, that according to our interpretation

$$F^0(d_n h_n) - 3F^0(s_n h_n) = \Sigma(Sn^{119}) - \Sigma(Sn^{117}) = -326 \text{ keV}, \quad (38)$$

where Σ denotes the sum

$$\Sigma = (E_{1/2} - E_{11/2}) + (E_{3/2} - E_{11/2}). \quad (38')$$

Inserting the value (38) in (37c) one arrives at

$$-F^0(s_n d_n) - 326 \text{ keV} > 0, \quad (39)$$

which is impossible, as the F^0's are positive by definition. But also if we assume that the $s_{1/2}$ shell is already filled in Sn^{121}, we come to a contradiction with the experimental facts, because in this case the difference $E(d_{3/2}) - E(h_{11/2})$ should decrease from Sn^{121} to Sn^{123} (and from this to Sn^{125}) by an amount equal to its increase from Te^{131}, Xe^{133}, Ba^{135} to Te^{133}, Xe^{135}, Ba^{137}. A glance at fig. 4 suffices to show the futility of this effort to preserve consistency.

In a similar manner consistency requires that $s_{1/2}$ will not be empty in the $d_{3/2}$ state of $Te^{125, 127, 129}$, but then we have, as far as the last two isotopes are concerned, the same disagreement with experiment as in Sn, and for Te^{125} this requirement is in contradiction to the qualitative explanation we have given to the $E(s_{1/2}) - E(d_{3/2})$ systematics in section 23.

It seems that the pure-configuration approximation cannot explain the $h_{11/2}$ systematics in the middle of the $h_{11/2}$ shell. Which configurations contribute to the states, and whether the simple mixing suggested by de Shalit and Goldhaber (section 8) is the only one of importance, cannot be answered without detailed calculations trying to adjust the experimental data. Perhaps such calculations will show that also the $E(s_{1/2}) - E(d_{3/2})$ systematics is governed by factors different from the various stabilization effects of section 23, as might be indicated by the suggested ground state $d_{3/2}$ of Xe^{127}.

26. The filling of the proton shells in the odd-Z nuclei. Mixed proton configurations

Though in the previous sections we assumed that all the protons in the Sn, Te, Xe and Ba isotopes were in $g_{7/2}$, it is evident from the systematics of the first excited 2^+ states of the even isotopes of Xe and Ba (section 22) and from the different behaviour of the $h_{11/2}$ lines in the different elements of fig. 4, that the proton configurations are rather mixed.

In the present section we shall consider the proton shell filling in the odd-Z nuclei. Although level schemes have been given for

many of these nuclei [33, 42, 43, 44, 32, 45, 46, 47]), spin and parity assignments are rather meager. Therefore, in assigning the odd-group configurations we shall have to rely mainly on the nuclear moments, about which much more is known (table 11) than about the moments of the odd-N nuclei in the same region.

TABLE 11

Ground State Configurations of Odd-Z Nuclei before $N = 82$

Nucleus	Z	N	$I\pm$ Ref.	μ	Ref.	Q	Ref.	Proton Shells $d_{5/2}$	$g_{7/2}$		Neutron Shells $s_{1/2}$	$d_{3/2}$	$h_{11/2}$	
Sb^{117*}	51	66	$5/2^+$ 74					1		a			2	
Sb^{121}		70	$5/2^+$ 49(d),33	3.360	49	—0.52	49	1		a			6	
Sb^{123}		72	$7/2^+$ 49(d),33	2.547	49	—0.67	49		1	a			8	
Sb^{125*}		74	$7/2^+$ 33						1	a			10	
I^{125*}	53	72	$5/2^+$ 33					1	2				8	
I^{127}		74	$5/2^+$ 49(d),33	2.808	49	—0.75	49	1	2	a			10	
I^{129*}		76	$7/2^+$ 49(d),33	2.617	49	—0.47	49		3				12	
I^{131*}		78	$7/2^+$ 49(d),33			—0.412	49		3		2		12	
I^{133*}		80	$7/2^+$ 33						3		2	2	12	
Cs^{131*}	55	76	$5/2^+$ 49(d),33	3.48	49			1	4				12	
Cs^{133}		78	$7/2^+$ 49(d),33	2.579	49	—0.003	75		5†		2		12	
Cs^{135*}		80	$7/2^+$ 49(d),33	2.727	49				5		2	2	12	
Cs^{137*}		82	$7/2^+$ 49(d),33	2.840	49				5		2	4	12	a
La^{139}	57	82	$7/2^+$ 49(d),45	2.778	49	0.3††	76	?	?		2	4	12	a
Pr^{141}	59	82	$5/2^+$ 49(d),33	3.9	49	—0.054	49	3	6	a?	2	4	12	a

† This state is really a mixture, as explained in the text.
†† According to Murakawa [70]) the value is 0.6.

From Sb^{121} to Sb^{123} the ground state changes from $d_{5/2}$ to $g_{7/2}$. This means that the $h_{11/2}$ shell, into which the additional neutron pair entered, favours g_p to d_p [21]). The next pair also entered $h_{11/2}$, which results in the increase of 173 keV of the excitation energy of the $d_{5/2}$ state from Sb^{123} to Sb^{125} [4]).

The ground state of I^{127} is $d_{5/2}$, and its quadrupole moment is negative, and even greater in absolute value than the single-$d_{5/2}$-proton moment of Sb^{121}. This determines its proton configuration as $d_{5/2}g^2_{7/2}$. The same conclusion is suggested also by the fact that the ground state of Sb^{125} is $g_{7/2}$, and therefore the ground state of I^{127}, if it has the same neutron configuration, cannot be $d^3_{5/2}$.

The ground state of I^{129} is $g_{7/2}$. If the last neutron pair entered the $h_{11/2}$ shell, which was seen above to prefer g_p to d_p, the con-

figuration may be only $g^3_{7/2}$, by a consistency comparison with I^{127}. The decrease of the absolute value of the quadrupole moment from I^{127} to I^{129} is perhaps connected with the change from a single proton moment to a three protons moment according to (24) †.

In I^{131} the quadrupole moment about equals that of I^{129}, which suggests that here too the proton configuration is $g^3_{7/2}$.

In Cs^{131} the ground state is $d_{5/2}$. If the neutron configuration is the same as in I^{129}, the only proton configuration allowed by consistency considerations is $d_{5/2}g^4_{7/2}$. Here, as in I^{127}, the tendency of the protons towards equal distribution among their shells outweighs the pull into the $g_{7/2}$ shell exerted by the $h_{11/2}$ neutrons.

In Cs^{133} the quadrupole is negative, but is about a hundred times smaller than the moment of a single $g_{7/2}$ proton as obtained from the Sb or the I nuclei. According to (26) such an almost vanishing moment may be obtained by mixing $d^2_{5/2}g^3_{7/2}$ and $g^5_{7/2}$ with about equal amplitudes. If (24) held exactly, the negative sign would mean that the coefficient of $d^2_{5/2}g^3_{7/2}$ in the mixture is a little larger. However, the deviations of the moments of $I^{127, 129, 131}$ from the formula are much larger than the whole value of the moment of Cs^{133}, and we should not attach too much significance to the sign of such a small moment. In table 11 we assigned the proton configuration of Cs^{133} as $g^5_{7/2}$ to agree with the treatment of the odd-N nuclei, where all the protons were assumed to be $g_{7/2}$. However, until additional quadrupole moments and energy level schemes are known in the I, Cs, La and Pr nuclei, nothing certain can be said about the constitution of the states.

Perhaps the small quadrupole moment of Pr^{141} is also connected with a mixture of $d^3_{5/2}g^6_{7/2}$ and $d_{5/2}g^8_{7/2}$, in which the share of the former is decisive.

CHAPTER 6. THE COMPETITION BETWEEN THE PROTON SHELLS $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$ AND THE COMPETITION BETWEEN THE NEUTRON SHELLS $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$, $1i_{13/2}$

27. General remarks

It might be expected that our approximation will give better results in the isotopes of Ir, Pt, Au, Hg, Tl and Pb, which are dealt

† According to de Shalit and Goldhaber the pull of the odd proton from $d_{5/2}$ to $g_{7/2}$ has already begun in I^{127}, whose ground state proton configuration contains an appreciable component of $(g^3_{7/2})_{5/2}$ which might explain the large deviation of its magnetic moment from the Schmidt line [21]).

with in this chapter. Here, both neutrons and protons approach magic numbers, and therefore the number of configurations which can interact strongly becomes quite small, and a pure-configuration approximation may be supposed to be a good one. Besides, the nuclei mentioned are among the heaviest, and therefore an approximation treating neutrons and protons separately will be better here than anywhere else. Likewise, a delta interaction may represent the facts better in these larger nuclei than in the smaller ones.

The shells mentioned in the caption of this chapter begin to be filled much earlier than in the above mentioned elements. However, there we find ourselves in the ("nuclear") rare-earth region, the investigation of which lies outside the scope of the present work (section 1).

The discussion of the odd-N nuclei will be rather fragmentary for lack of sufficient experimental data.

28. Proton shell filling in the odd-Z nuclei

The sign of the quadrupole moments of $Ir^{191, 193}$ and of Au^{197} shows that they have three $d_{3/2}$ protons, although at least in Ir the $h_{11/2}$ shell is certainly not yet filled.

In order to know the proton configurations we have to answer the question whether the $s_{1/2}$ shell is full or not. The much larger pairing energy of the $s_{1/2}$ shell (table 3) suggests that it becomes filled before the $d_{3/2}$ shell begins to be filled with pairs (as happened for the neutrons in Te^{131}, Xe^{133} and Ba^{135}). But since there are also other factors governing shell filling, this argument cannot be taken as decisive.

The answer for the Au nuclei comes from a consideration of the systematics of $E(s_{1/2})-E(d_{3/2})$ in the Au and the Tl nuclei fig. 6). The energy differences are approximately linear in both elements, with almost equal slopes, which leads to the assumption that the neutron pairs are added in the same shell from one isotope to another in both elements (at least up to Tl^{201} or Tl^{203}), and that the $s_{1/2}$ state is obtained from the $d_{3/2}$ state by a proton transition in the same direction. The $s_{1/2}$ state in Tl is obtained from the $d_{3/2}$ state by a proton transition from $s_{1/2}$ to $d_{3/2}$ (table 12), and only the ground state configuration $s^2_{1/2}d^3_{3/2}h^{10}_{11/2}$ enables the transition to take the same direction in Au, too. We see that here the decisive factor influencing shell filling is the equilibrium between

the number of protons in the $h_{11/2}$ shell on the one hand and the number of protons in the $s_{1/2}$ and the $d_{3/2}$ shells on the other and not the large pairing energy of the $h_{11/2}$ shell, which was the case with the corresponding odd-N nuclei.

The change of the ground state spin from 3/2 in Au to 1/2 in

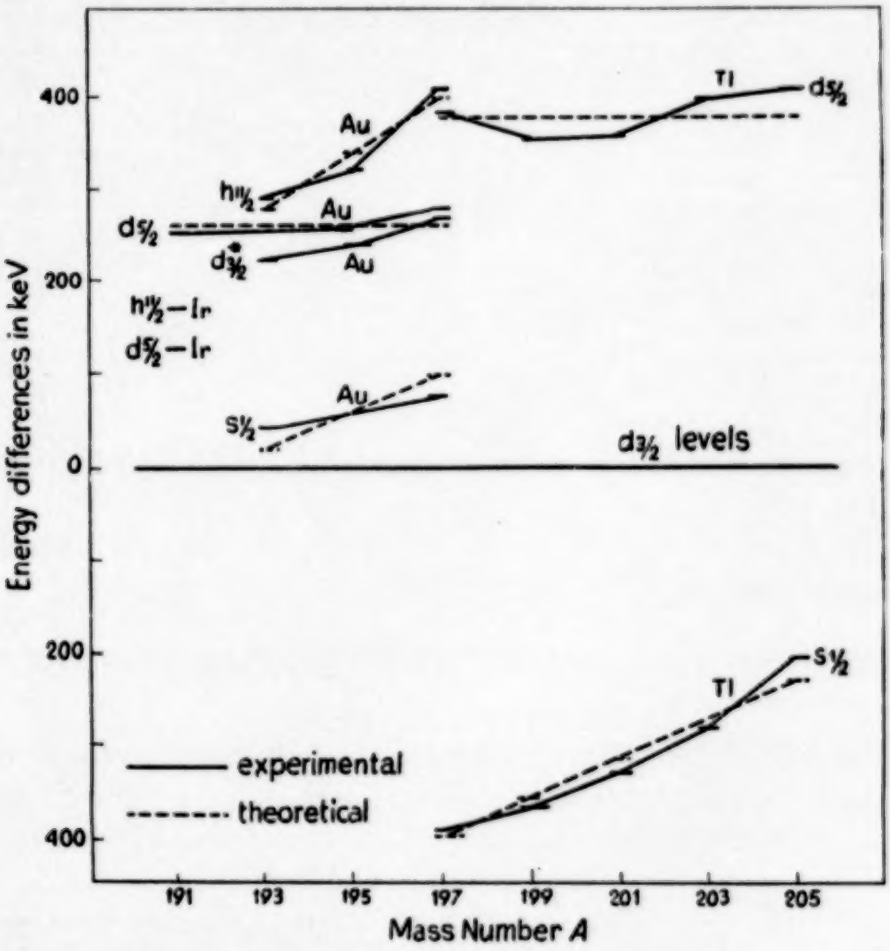

Fig. 6. Level systematics of odd-Z nuclei before $Z = 82$.
References: Ir^{191} (95); $Au^{191, 193, 195}$ (77); Au^{197} (33); $Tl^{197, 199, 201, 203}$ (59); Tl^{205} (96).

Tl shows that h_p prefers d_p to s_p, which was also the case in the odd-N nuclei, and which is in accordance with table 2.

A word should be said here about the shell into which the neutrons enter. If we regard the $s_{1/2}$ graph of Tl as a straight line up to Tl^{205}, eight neutrons enter this shell, beginning with Tl^{197}, and thereore it can only be $i_{13/2}$. However, the linearity of the $s_{1/2}$-Tl graph is not so certain, as will be explained in the following section. The decrease of the difference $E(s_{1/2}) - E(d_{3/2})$ in the heavier isotopes

TABLE 12

Ground State Proton Configurations of Odd-Z Nuclei before $Z = 82$, $N = 126$

Nucleus	Z	N	$I\pm$	Ref.	μ	Ref.	Q	Ref.	Proton Shells $s_{1/2}$	$d_{3/2}$	$h_{11/2}$
Ir^{191}	77	114	$3/2^+$	49(d),33	0.17	49	1.5	49	?	3	?
Ir^{193}		116	$3/2^+$	49(d),33	0.17	49	1.5	49	?	3	?
Au^{191*}	79	112	$3/2^+$	77					2	3	10
Au^{193*}		114	$3/2^+$	77					2	3	10 b
Au^{195*}		116	$3/2^+$	33					2	3	10 b
Au^{197}		118	$3/2^+$	49(d),33	0.13	49	0.56	49	2	3	10 b
Au^{199*}		120	$3/2^+$	78(d),33					2	3	10
Tl^{197*}	81	116	$1/2^+$	59					1	4	12 a
Tl^{199*}		118	$1/2^+$	59					1	4	12 a
Tl^{201*}		120	$1/2^+$	59					1	4	12 a
Tl^{203}		122	$1/2^+$	49(d),33	1.611	49			1	4	12 a
Tl^{205}		124	$1/2^+$	49(d),79	1.627	49			1	4	12 a
Tl^{207*}		126	$1/2^+$	79					1	4	12 a

of Au and Tl shows that the neutron shell which is being filled prefers s_p to d_p.

The proton configurations of table 12 are those deduced from the above considerations.

29. A QUANTITATIVE TREATMENT OF THE ENERGY SYSTEMATICS OF Au AND Tl

A glance at fig. 6 suffices to show that the agreement between the experimental data and the linear formulas will be fairly good. Before proceeding we should complete the discussion of the transitions by which the excited states are formed.

A remarkable fact concerning the $d_{5/2}$ states in Au is the constancy of their excitation energy [48]). In our approximation this cannot be otherwise if the $d_{5/2}$ state is produced by a transition of a proton from the already full $d_{5/2}$ shell to the $d_{3/2}$ shell. This transition does not change the total number of 2d protons, and we know that the interaction of these protons with other shells of nucleons with $J = 0$ is independent of their j-value. Therefore, the excitation energy of this transition equals the internal energy change of the 2d protons, which is

$$A_{5/2} - A_{3/2} = (\tfrac{5}{2}) \cdot \zeta^p(2d),$$

where $\zeta^p(2d)$ is the positive spin-orbit parameter of the proton shells 2d.

One problem arising is the inconstancy of the doublet splitting $E(d_{3/2})-E(d_{5/2})$ in the Tl isotopes, and another much more disturbing one is the distinct difference between the average splittings in the Tl and the Au nuclei. It might perhaps be argued that the excitation energy of the $d_{3/2}$ states in the Tl isotopes is only slightly less than that of the first excited states in the even Hg nuclei constituting the cores of the Tl nuclei (fig. 8, section 32), and that the $d_{5/2}$ levels are still much higher. Therefore, the single-particle approximation is already invalid for them, and the interaction with other states might be responsible for the irregular variations in the magnitude of the doublet splitting. This interaction might also be responsible for the increase in the doublet splitting in Tl by lowering its $d_{3/2}$ more than the $d_{5/2}$ states; but it is also worth mentioning that the increase in the splitting from Ir^{191} to the Au isotopes and from these to the average splitting of the Tl isotopes is about the same, indicating, perhaps, a linear dependence of the $\zeta^p(2d)$ upon the proton number. It is desirable to discover the excited states in additional Ir isotopes in order to decide this.

The $h_{11/2}$ states in Au may be excited either by a transition of the odd proton from the $d_{3/2}$ shell to the $h_{11/2}$ shell, or vice versa. In the first case the experimental data would indicate that the neutron shell into which the neutrons enter in the heavier isotopes prefers d_p to h_p, and in the second case the preference would be the opposite. Which of the two possibilities holds in reality will be known after the systematics of the $h_{11/2}$ levels in the Tl nuclei is discovered, since in them the $h_{11/2}$ state may be excited only by a proton transition from the $h_{11/2}$ shell to the $s_{1/2}$ shell. The change of the energy difference $E(d_{3/2})-E(h_{11/2})$ from the lighter to the heavier Tl isotopes will show which proton shell is preferred by the neutrons, and therefore in what way the $h_{11/2}$ states in Au are formed; however, the reliability of the last conclusion remains questionable, because the $h_{11/2}$ states may be much higher than the first excited states of the Hg "cores", and then the single-particle approximation may be inapplicable to them.

The excited $d_{3/2}$ state (which will be designated by $d^*_{3/2}$) may be obtained from the ground state by a transition of a proton pair from the $d_{3/2}$ or from the $s_{1/2}$ shell to the $h_{11/2}$ shell, or by a transition of a neutron pair from shell to shell, or by a comination of such transitions. The fact that the excitation energy $E(d_{3/2})-Ed^*_{3/2})$

is a linear function of N suggests that the transition is the same in the three Au isotopes and that the neutrons are added in them in the same shell. A measurement of the quadrupole moment of the $d^*_{3/2}$ states would supply partial information concerning the proton transitions. At this stage, by considering the slopes of the graphs in fig. 6, it may only be said that if our approximation is good, the transition cannot be due to protons only. For example, it cannot be a transition of a proton pair from the $d_{3/2}$ to the $h_{11/2}$ shell, since in this case the absolute value of the slope of the $d^*_{3/2}$ line would have to be twice the absolute value of the (average) slope of the $h_{11/2}$ line †. Similarly, the possibility of a $s_{1/2}$ proton pair transition can be seen to be precluded by the facts. In conclusion, before we have more complete information about the neutron shell filling and the parameters appearing in the formulas of the odd-N nuclei systematics, we cannot say how the $d^*_{3/2}$ states are produced.

The following least-square calculation is intended merely to find quantitatively the degree of agreement between the linear graphs of fig. 6 and the linear formulas of our approximation. The $d^*_{3/2}$ states are not included, because we do not know now if their agreement, which would be good, is real. Also, we did not assign the same parameter of spin-orbit interaction to the Au and Tl nuclei, as it is obvious that this will not agree with the facts. We assumed that the $s_{1/2}$ line of Tl is straight, and therefore that the neutrons, which determine the slopes of the lines of fig. 6, are in $i_{13/2}$. As we do not know the exact neutron configurations in the Au and the Tl nuclei we could define in the usual way only the parameters giving the slopes of the lines; the others were only empirically defined. We used the parameters

$$A = E_{1/2}(\mathrm{Au}^{193}) - E_{3/2}(\mathrm{Au}^{193}) \tag{40a}$$

$$B = E_{1/2}(\mathrm{Tl}^{197}) - E_{3/2}(\mathrm{Tl}^{197}) \tag{40b}$$

$$C = F^0(i_n d_p) - F^0(i_n s_p) \tag{40c}$$

$$D = E_{3/2}(\mathrm{Au}^{193}) - E_{11/2}(\mathrm{Au}^{193}) \tag{40d}$$

$$G = \pm(F^0(i_n d_p) - F^0(i_n h_p)) \tag{40e}$$

(where the sign is + or − according to the $h_{11/2}$ state being formed by a proton transition from the $d_{3/2}$ shell to the $h_{11/2}$ shell or by the opposite transition, respectively)

† We must confine ourselves to absolute values, as we do not know the direction of the transition producing the $h_{11/2}$ states.

$$Z = A_{5/2}(\mathrm{Au}) - A_{3/2}(\mathrm{Au}) = (\tfrac{5}{2})\zeta^{\mathrm{p}}(2\mathrm{d})_{\mathrm{Au}} \qquad (40\mathrm{f})$$

$$Z' = A_{5/2}(\mathrm{Tl}) - A_{3/2}(\mathrm{Tl}) = (\tfrac{5}{2})\zeta^{\mathrm{p}}(2\mathrm{d})_{\mathrm{Tl}}. \qquad (40\mathrm{g})$$

The results of the calculation are given in table 13. Actually there are four independent calculations, and we give the mean error of each calculation separately. The theoretical graphs of fig. 6 were obtained from table 13.

We would like to remark that according to the discussion in section 31 it may very probably happen that the last neutron pair (or pairs) in the heavier Tl isotopes does not enter the $i_{13/2}$ but the $f_{5/2}$ shell. In this case there should be a change in the direction of the $s_{1/2}$ line of Tl, which is perhaps suggested in fig. 6 after Tl^{201} or Tl^{203}. Adopting such an interpretation would of course improve the agreement between theory and experiment over that of table 13. However, we did not want to assume this before something more definite is known about neutron shell filling.

30. A THEORETICAL TREATMENT OF THE $p_{3/2}$—$p_{1/2}$ COMPETITION, AND THE ASSIGNMENT OF THE PARTIAL 3p CONFIGURATIONS IN THE ODD-N NUCLEI

Since the interaction of the neutrons in the 3p shells with the nucleons in the other shells is the same whether the former are in the $p_{3/2}$ or in the $p_{1/2}$ shell, the distribution between the $p_{3/2}$ and the $p_{1/2}$ shells of a given number of 3p neutrons will depend solely upon this number.

The energy differences between the various $p_{3/2}^{n-k}\, p_{1/2}^{k}$ configurations are given in our approximation in terms of the two parameters

$$a = (\tfrac{3}{2})\zeta^{\mathrm{n}}(3\mathrm{p})$$

and

$$f = F^0(\mathrm{p_n\, p_n}).$$

Using the formulas of chapter 1, one obtains for the differences between the lowest levels of the 3p configurations the values

$$E(\mathrm{p}_{3/2}) - E(\mathrm{p}_{1/2}) = a_{3/2} - a_{1/2} = a \qquad (41\mathrm{a})$$

$$E(\mathrm{p}_{3/2}^2) - E(\mathrm{p}_{1/2}^2) = 2a + f \qquad (41\mathrm{b})$$

$$E(\mathrm{p}_{3/2}^3) - E(\mathrm{p}_{3/2}^2\mathrm{p}_{1/2}) = a - f \qquad (41\mathrm{c})$$

$$E(\mathrm{p}_{3/2}^3) - E(\mathrm{p}_{3/2}\mathrm{p}_{1/2}^2) = 2a \qquad (41\mathrm{c}')$$

$$E(\mathrm{p}_{3/2}^4) - E(\mathrm{p}_{3/2}^2\mathrm{p}_{1/2}^2) = 2a - f \qquad (41\mathrm{d})$$

$$E(\mathrm{p}_{3/2}^4\mathrm{p}_{1/2}) - E(\mathrm{p}_{3/2}^3\mathrm{p}_{1/2}^2) = a, \qquad (41\mathrm{e})$$

TABLE 13

Comparison of the Au and Tl Energy Level Systematics with the Theory

Nucleus	Parameters			Exp.	Cal.	Diff.
I. $E(s_{1/2})-E(d_{3/2})$						
	A	B	C			
Au^{193}	1			— 38	— 18	—20
Au^{195}	1		2	— 61	— 59	— 2
Au^{197}	1		4	— 77	—100	23
Tl^{197}		1		388	396	— 8
Tl^{199}		1	2	367	355	12
Tl^{201}		1	4	331	314	17
Tl^{203}		1	6	280	273	7
Tl^{205}		1	8	205	232	—27
mean deviation						±17
mean error						±21
II. $E(d_{3/2})-E(h_{11/2})$						
	D	G				
Au^{193}	1			291	281	10
Au^{195}	1	2		319	340	—21
Au^{197}	1	4		409	399	10
mean deviation						±15
mean error						±25
III. $E(d_{3/2})-E(d_{5/2})$ for Au						
	Z					
Au^{191}	1			253	263	—10
Au^{193}	1			259	263	— 4
Au^{195}	1			262	263	— 1
Au^{197}	1			279	263	16
mean deviation						±10
mean error						±11
IV. $E(d_{3/2})-E(d_{5/2})$ for Tl						
	Z'					
Tl^{197}	1			386	383	3
Tl^{199}	1			355	383	—28
Tl^{201}	1			362	383	—21
Tl^{203}	1			400	383	17
Tl^{205}	1			410	383	27
mean deviation						±21
mean error						±24

Most Probable Values of the Parameters

A	— 18 keV	G	30 keV
B	396 keV	Z	263 keV
C	— 21 keV	Z'	383 keV
D	281 keV		

which shows that for one, two and five 3p neutrons as many as possible will enter the $p_{3/2}$ shell, which is but natural for *jj*-coupling. For three and four neutrons the conclusion is a little less definite as it depends on the ratio of *f* to *a*. However, if the actual situation approximates the *jj*-coupling, *f* should be much smaller than *a*, and therefore three and four neutrons will also all be in the $p_{3/2}$ shell.

Experimentally, *a* equals the large doublet splitting of 900 keV of Pb^{207} (fig. 7), and this confirms the above reasoning, as certainly the value of *f* cannot be so high. Another confirmation of the result obtained is the positive quadrupole moment of Hg^{201}

TABLE 14

Ground State Configurations of Odd-*N* Nuclei before $Z = 82$, $N = 126$

Nucleus	Z	N	$I\pm$	Ref.	μ	Ref.	Q	Ref.	Proton Shells $s_{1/2}$	$d_{3/2}$	$h_{11/2}$		Neutron Shells $f_{5/2}$	$p_{3/2}$	$p_{1/2}$	$i_{13/2}$	
Os^{187}	76	111	1/2	70(d)	0.12	70			?	?	?		?	?	?	?	a
Os^{189}	76	113	3/2†	(49(d)	0.65	49	2	49	?	?	?		?	?	?	?	
Hg^{193*}	80		3/2⁻	77					2	2	12		?	?	?	?	
Os^{191*}	76	115	9/2⁻†	80					?	?	?		?	?	?	?	
Pt^{193*}	78		3/2⁻?††						2	2	10	e	?	3		?	
Hg^{195*}	80		3/2⁻	77					2	2	12		?	3		?	
Pt^{195}	78	117	1/2⁻	49(d),33	0.601	49			2	2	10	e	?	4	1	?	
Hg^{197*}	80		1/2⁻	49(d),33	0.52	81			1	2	12		?	4	1	?	
Pt^{197*}	78	119	1/2⁻	33					2	2	10	e	?	4	1	?	
Hg^{199}	80		1/2⁻	49(d),33	0.499	49			2	2	12	e	?	4	1	?	
Hg^{201}	80	121	3/2⁻	49(d),82	—0.554	49	0.45	70	2	2	12	e	?	3		?	
Hg^{205*}	80	125	1/2⁻	79					2	2	12		6	4	1	14	a
Pb^{207}	82		1/2⁻	49(d),33	0.584	49			2	4	12	a	6	4	1	14	a

† Not a single-particle state.

†† We suggest a $p_{3/2}$ state from similarity considerations: there is a change of direction in the $i_{13/2}$ line of the Hg nuclei at Hg^{197} (fig. 7) accompanied by a change of the ground state spin from $p_{3/2}$ to $p_{1/2}$. There is a similar change at Pt^{195}, which brought us to the above suggestion.

with $j = 3/2$, indicating a $p^3_{3/2}$ configuration, and therefore that three 3p neutrons will all be in the $p_{3/2}$ shell. Here we are sure that the configuration is not $p^3_{3/2}p^2_{1/2}$, because for five 3p neutrons the configuration $p^4_{3/2}p_{1/2}$ is more stable.

The above conclusions could also be obtained directly, and for

more general types of forces, from figures 4^{11}, 5^{11}, 1^{13} and 3^{13} of Condon and Shortley [7]).

This determines unequivocally the configuration of the 3p neutrons in all the ($J = 1/2$)-odd-N nuclei as $p^4_{3/2}p_{1/2}$, since with either three or one 3p neutrons the J would be 3/2 in the ground state. In addition to this the partial 3p configuration of Hg^{201} is $p^3_{3/2}$ (table 14).

In order to completely assign the neutron configurations in the odd-N nuclei, one still has to know the distribution of the neutrons in the shells other than 3p. As a spin 5/2 has not been found with certainty in any of the Os, Pt, Hg and Pb nuclei [49]), it is usually assumed that shell $2f_{5/2}$ is already full in these nuclei [50, 1, 51]). However, we shall show in the following section that we cannot adopt this assumption.

31. M4 TRANSITIONS IN Pt, Hg AND Pb. A PROBABLE DELAYED FILLING OF THE $f_{5/2}$ NEUTRON SHELL

Looking at the graphs of the $i_{13/2}$ levels in fig. 7 one observes undistorted parallelograms of the kind described in section 17, in the nuclei $Pt^{195, 197}$, $Hg^{197, 199}$ and $Pb^{197, 199, 201, 203}$. According to section 17 this might mean that the proton configuration is the same in the various isotopes of each element, and that the $i_{13/2}$ states are obtained from the $f_{5/2}$ states by the same neutron transition in all these nuclei. This also implies that the neutrons are added in the same shell in the various isotopes of each element, which is also more directly suggested by the linearity of the $i_{13/2}$ line of the Pb nuclei.

However, in spite of the linearity of the systematics we did not find a simple explanation for it. On the assumption that the neutron shell $f_{5/2}$ is already closed, the neutron configurations of $Pt^{195, 197}$ and $Hg^{197, 199}$ are uniquely determined by the discussion in the preceding section, resulting in an impossibility of a consistent interpretation of the $E(f_{5/2})-E(i_{13/2})$ systematics, as will be expounded below. On the other hand we had no suggestion as to how to assign the neutron configurations, as (disregarding Pb^{207}) complete level schemes are known only for Pt and Hg nuclei quite removed from $N = 126$, in which there are many possibilities to obtain given J states.

How does inconsistency arise? If the $f_{5/2}$ shell is filled, the neutron pairs in the heavier isotopes are added in $i_{13/2}$. Then the

fact that $E(f_{5/2})-E(i_{13/2})$ increases with N shows that the number of neutrons in the $i_{13/2}$ shell is larger in the $i_{13/2}$ state than in the $f_{5/2}$ state (this appears very clearly from the considerations in sections 9 and 10). This requirement determines uniquely the con-

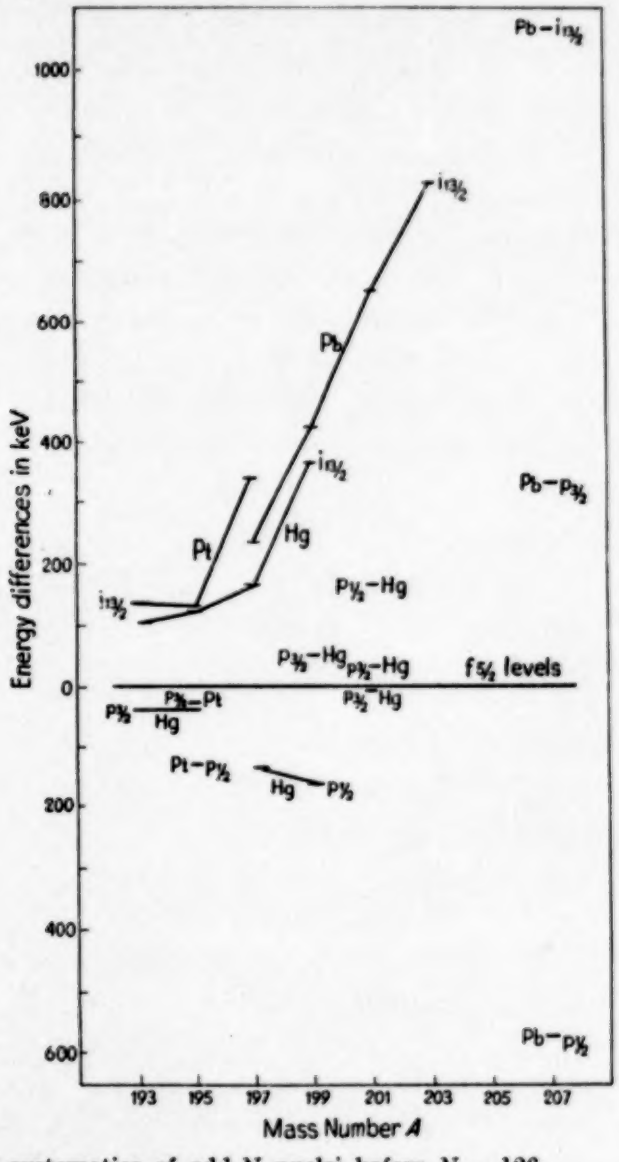

Fig. 7. Level systematics of odd-N nuclei before $N = 126$.
References: Pt^{193} (97); Pt^{195} (33); Pt^{197} (4); $Hg^{193\ 195}$ (77); Hg^{197} (33); Hg^{199} (60); Hg^{201} (82); Pb^{197} (59,53); Pb^{199} (59); Pb^{201} (98); Pb^{203} (53); Pb^{207} (99).

figurations of the excited $f_{5/2}$ and $i_{13/2}$ states, as shown below, but the determination is found to be inconsistent.

With a closed $f_{5/2}$ shell the $p_{1/2}$ ground state configuration of an odd-N nucleus may only be $f^6_{5/2}p^4_{3/2}p_{1/2}i^n_{13/2}$, the $i_{13/2}$ configuration may be either

$$f^6_{5/2}\,p^4_{3/2}\,i^{n+1}_{13/2} \qquad \text{(a)}$$

or

$$f^6_{5/2}\,p^4_{3/2}\,p^2_{1/2}\,i^{n-1}_{13/2} \qquad \text{(b)}$$

and the $f_{5/2}$ configuration may only be

$$f^5_{5/2}\,p^4_{3/2}\,p^2_{1/2}\,i^n_{13/2}, \qquad \text{(a')}$$

if we restrict ourselves to single-particle transitions. Only the combination (aa') satisfies the above requirement, which determines the configurations uniquely, as asserted before.

But here the question arises why a proton pair should remain in the $p_{1/2}$ shell in the $f_{5/2}$ states of Pt^{195}, Hg^{197} or Pb^{199}, while in the $i_{13/2}$ states the $p_{1/2}$ shell is empty even in the heavier isotopes of the same elements, where the ensuing loss in the interaction between the $p_{1/2}$ and the $i_{13/2}$ shells is much larger than in the lighter isotopes. Writing explicitely the stability conditions for the above mentioned nuclei against a transition of a neutron pair from the $p_{1/2}$ to the $i_{13/2}$ shell †:

$$f_{5/2}(Hg^{197}, Pb^{199}):\ 2(a_{1/2}-a_{13/2}) + (\pi_{1/2}-\pi_{13/2}) + 6F^0(p_n i_n) + 5(F^0(f_n p_n) - F^0(f_n i_n)) > 0 \qquad \text{(43a)}$$

$$i_{13/2}(Hg^{199}, Pb^{201}):\ 2(a_{1/2}-a_{13/2}) + (\pi_{1/2}-\pi_{13/2}) + 7F^0(p_n i_n) + 6(F^0(f_n p_n) - F^0(f_n i_n)) < 0 \qquad \text{(43b)}$$

$$i_{13/2}(Pb^{203}):\ 2(a_{1/2}-a_{13/2}) + (\pi_{1/2}-\pi_{13/2}) + 9F^0(p_n i_n) + 6(F^0(f_n p_n) - F^0(f_n i_n)) < 0, \qquad \text{(43c)}$$

we obtain from (43a) and (43b)

$$F^0(p_n i_n) + F^0(f_n p_n) - F^0(f_n i_n) < 0 \qquad \text{(43d)}$$

and from (43a) and (43c)

$$3F^0(p_n i_n) + F^0(f_n p_n) - F^0(f_n i_n) < 0. \qquad \text{(43e)}$$

But according to the present interpretation of the levels

$$F^0(p_n i_n) - F^0(f_n i_n) = (E_{5/2}(Hg^{199}) - E_{1/2}(Hg^{199})) - (E_{5/2}(Hg^{197}) - E_{1/2}(Hg^{197})) = -25 \text{ keV} \qquad \text{(44a)}$$

$$F^0(p_n i_n) = (E_{1/2}(Hg^{199}) - E_{13/2}(Hg^{199})) - (E_{1/2}(Hg^{197}) - E_{13/2}(Hg^{197})) = 228 \text{ keV}. \qquad \text{(44b)}$$

Inserting the values (44a, b) in (43d, e) one obtains

† For the sake of simplicity we do not write the effect of the proton shells on the transition, which is of no consequence to the final result, as the protons influence the transition in the same way for the $f_{5/2}$ and for the $i_{13/2}$ states.

$$F^0(f_n p_n) - 25 \text{ keV} < 0 \tag{45a}$$

$$F^0(f_n p_n) + 431 \text{ keV} < 0; \tag{45b}$$

(45a) is very improbable according to table 2 and to the experience we have gained so far with the F^0's values, and (45b) cannot hold in principle.

Difficulties also arise in connection with the competition between the neutron shells $p_{3/2}$ and $p_{1/2}$ if we assume that $f_{5/2}$ is already closed. To be specific, the change of the ground state spin from $p_{3/2}$ in Hg^{195} to $p_{1/2}$ in Hg^{197} and again to $p_{3/2}$ in Hg^{201} leads to contradictions, and the very different magnitudes of the doublet splitting in the various nuclei of fig. 7 result in a disagreement between theory and experiment, when a quantitative explanation of the systematics of fig. 7 is attempted.

We shall now add some remarks which might more directly indicate that the $f_{5/2}$ shell is not yet closed in the nuclei considered above.

In the first place, according to recent investigations [52, 53]) the ground states of $Pb^{197, 199, 201, 203}$ are probably all $f_{5/2}$, as no γ-rays were found in cascade with the M4 γ's leading from the $i_{13/2}$ states to the $f_{5/2}$ states. β-deacy investigations lead to a similar conclusion concerning Pb^{203} [54, 55, 56]) and also Hg^{203} [56]).

The second remark concerns the g-factor 0.054 of the excited state 4^+ of Pb^{204} [57]). As is known, the g-factor of a group of equivalent nucleons is equal to the g-factor of each single nucleon, and the above mentioned experimental value is much nearer to the 0.56 g-factor of $f_{5/2}$ neutrons than to the -2.95 value of $i_{13/2}$ ones. Therefore, the shell responsible for this 4^+ state is probably $f_{5/2}$. Though it must be admitted that the magnetic moment calculated with this g-factor does not fall within the allowed zone of $j = l - \frac{1}{2}$ [58]).

Finally, there are no low lying 6^+ levels in the even Pb isotopes [59]). These should be rather close to the 4^+ states according to (5), provided they may be excited at all within the shell. Their absence might indicate that they cannot really be excited, and, therefore, that the shell responsible for the 2^+ and 4^+ states cannot be $i_{13/2}$ but might be $f_{5/2}$. Moreover, according to table 3 the $E(0^+) - E(2^+)$ separation should be about 1340 keV for the $i_{13/2}$ shell and about 760 keV for the $f_{5/2}$ shell. The experimental separa-

tion of 899 keV in Pb^{204}, for example, is much nearer to the $f_{5/2}$ shell value.

32. A QUALITATIVE TREATMENT OF THE SYSTEMATICS OF THE EVEN ISOTOPES OF Pt, Hg AND Pb AND OF THE PROTON CONFIGURATIONS OF Pt AND Hg

A glance at fig. 8 shows that the behaviour of the excited states of the even Pt, Hg and Pb nuclei is to a good approximation represented by the formulas of section 11. There are

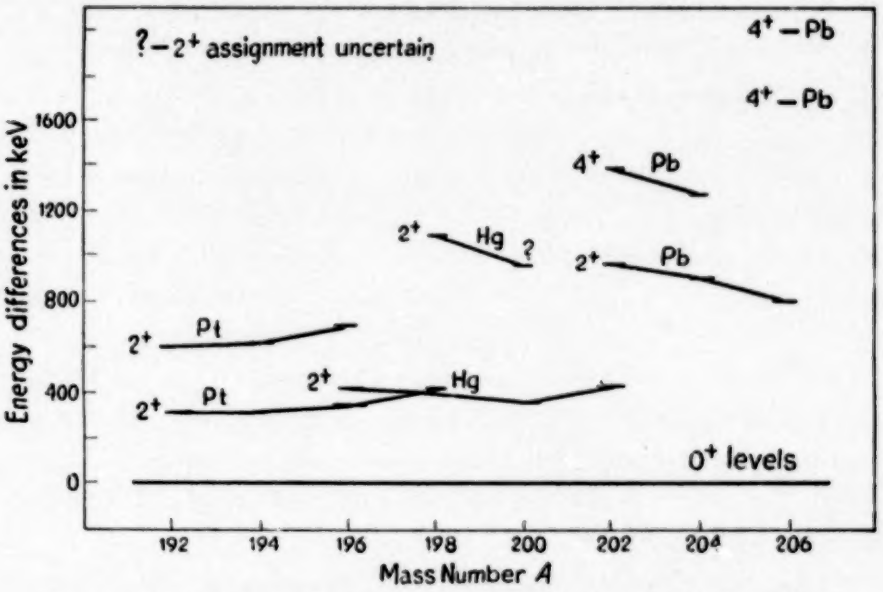

Fig. 8. 0^+, 2^+ and 4^+ levels of the even isotopes of Pt, Hg and Pb. References: Pt^{192} (77); Pt^{194} (100); Pt^{196} (89); Pt^{198} (101); $Hg^{196, 198}$ (33); $Hg^{200, 202}$ (60); $Pb^{202, 204}$ (59); Pb^{206} (27).

groups of 2^+ states of approximately constant excitation energies, each plausibly corresponding to a pair $j^2(2)$ of a different shell. Also the ratio $(E(0^+)-E(4^+))/(E(0^+)-E(2^+))$ in the Pb nuclei is much closer to the value 1.2 it should have according to table 3 than to the 3.3 value it should have in the Bohr model.

In the Pb nuclei it is certain that the excited states are formed by neutrons, as the proton number is magic. We have already seen in the preceding section that the 4^+ and the 2^+ states probably belong to the $f_{5/2}$ neutron shell.

In the Hg nuclei we have a group of 2^+ states (to be accurate, the J-assignment in Hg^{200} is doubtful [60])) corresponding to the 2^+ states of Pb, which it is but natural to assign to the same neutron shell. The first excited 2^+ state of Hg is then plausibly attributed

to a proton shell, as it does not appear in the $Pb^{202, 204}$ nuclei with the same numbers of neutrons.

The first excited state 2^+ in the Pt nuclei seems to belong to the same shell as the first excited state 2^+ in the Hg nuclei. However, the second excited states 2^+ of the Pt nuclei do not appear in the Hg nuclei with the same neutron numbers, and it is therefore plausible to attribute them to a second proton shell, having a higher excitation energy, which is already closed in Hg, and the corresponding 2^+ state cannot, therefore, be excited in the Hg nuclei. The proton shells available for excitation to 2^+ states are $d_{3/2}$ and $h_{11/2}$, and looking at table 3 one has no doubts that the shell with higher excitation energy is $h_{11/2}$. Therefore, the proton configuration is $s^2_{1/2}d^3_{3/2}h^{10}_{11/2}$ in Pt and $s^2_{1/2}d^3_{3/2}h^{12}_{11/2}$ in Hg. Evidently, this concerns only those Pt and Hg nuclei whose excited states appear in fig. 8 and correspond to the arguments of the last paragraphs.

It is worth mentioning that the regular variations of the differences $E(0^+)-E(2^+)$ in the Pt, Hg and Pb nuclei of fig. 8 suggest that the states considered are not so pure. A fact which might perhaps be explained by a simple interaction of the type suggested in section 8 is the parallel run of the 2^+ and 4^+ lines in $Pb^{202, 204}$. According to (15c) the interaction element between the two 2^+ states of seniority 2 is the same as between the two 4^+ states of the same seniority, obtained from each other by a transition of a pair $j^2(0)$ from shell to shell. The distance between the diagonal elements is also the same for the 2^+ states as for the 4^+ states, and therefore the lower 2^+ and 4^+ states are depressed by the same amount by the interaction (15c). However, the depression will depend upon the number of nucleons. Therefore, if the 2^+ and the 4^+ lines were parallel to the 0^+ line without the interaction, they will no longer be parallel to it, but will remain parallel to each other, after the operation of the interaction.

If interaction exists, the configurations are mixed, and one cannot usually attribute a definite state to a definite configuration. Therefore, the above configuration assignment for the Pt and the Hg protons should be taken with reservation, until a calculation taking into account the interaction will definitely determine the composition of the states in the even nuclei, or until additional data on the odd-N nuclei will enable us to assign their configurations in a more direct manner.

I am grateful to Professor Racah for suggesting this subject to me, and for continuous encouragement. The work benefited much from stimulating discussions with him and also with Drs. Igal Talmi, Amos de Shalit and Gideon Rakavy. And I also want to thank Dr. Ingmar Bergström, from the Nobel Institute for Physics, and Mr. Marcel Schmorak, who is now working at the same Institute, for information concerning the Pb nuclei prior to its publication.

Note added in proof: It is worth mentioning that the formulas remain linear also under no assumption whatsoever concerning the range of the nuclear forces, as long as the seniority remains a good quantum number, and the low-lying states of the even and the odd groups of nucleons are characterized by seniority 0 and 1 respectively.

The expression for the interaction between different shells (sections 6 and 7) is practically unaffected by such a generalization. One has only to stop at formulae (6) and (9), which is equivalent to replacing the F^0's by I^0's in the final formulas.

On the other hand, the expression for the energy of a group of equivalent nucleons (section 5) is radically changed, becoming a quadratic function of the numbers of nucleons, instead of linear as before (ref. [13]), eqs. (44a, b)). Using the language of spectroscopy, this is equivalent to the inclusion of a scalar interaction between the nucleons, which is absent with forces possessing the pairing property[19]) (like delta-forces).

However, this change does not affect the linearity of the energy differences (18a, b), as the energy expression still remains a quadratic function of the numbers of nucleons, with coefficients depending on the shells, but not on the J-values of the states.

References

1) P. F. A. Klinkenberg, Revs. Modern Phys. **24** (1952) 63
2) M. G. Mayer, Phys. Rev. **78** (1950) 16
3) A. Bohr and B. R. Mottelson, Dan. Mat-Fys. Medd. **27** (1953) No. 16
4) M. Goldhaber and R. D. Hill, Revs. Modern Phys. **24** (1952) 179
5) L. Rosenfeld, Physica **17** (1951) 461
6) A. Bohr, Rotational States of Atomic Nuclei (Munksgaards Forlag, Copenhagen, 1954)
7) E. U. Condon and G. H. Shortley, The Theory of Atomic Spectra (Cambridge University Press, Cambridge, 1935) (referred to as TAS in the present work)
8) G. Racah, Phys. Rev. **63** (1943) 367 (referred to as III in the present work)
9) M. G. Mayer, Phys. Rev. **78** (1950) 22
10) I. Talmi, Helv. Phys. Acta **25** (1952) 185
11) A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. A **215** (1952) 120
12) B. H. Flowers, Proc. Roy. Soc. A **215** (1952) 398
13) G. Racah, L. Farkas Memorial Volume, (Research Council of Israel, Jerusalem, 1952) p. 294
14) G. Racah, Bull. Res. Counc. of Israel **3** (1954) 290
15) M. H. L. Pryce, Proc. Phys. Soc. A **65** (1952) 773
16) I. Talmi, Phys. Rev. **90** (1953) 1001
17) A. de Shalit, Phys. Rev. **91** (1953) 1479

18) L. Rosenfeld, Nuclear Forces (North Holland Publ. Co., Amsterdam, 1948)
19) G. Racah and I. Talmi, Physica **18** (1952) 1097
20) G. Racah, Phys. Rev. **62** (1942) 438 (referred to as II in the present work)
21) A. de Shalit and M. Goldhaber, Phys. Rev. **92** (1953) 1211
22) G. Racah, Physica **16** (1950) 651
23) C. Schwartz and A. de Shalit, Phys. Rev. **94** (1954) 1257
24) C. Schwartz, Phys. Rev. **94** (1954) 95
25) L. G. Elliott, R. L. Graham, J. Walker and J. L. Wolfson, Phys. Rev. **93** (1954) 356
26) L. G. Elliott, unpublished work quoted by G. E. Tauber, Phys. Rev. **99** (1955) 176
27) D. E. Alburger and M. H. L. Pryce, Phys. Rev. **95** (1954) 1482
28) O. E. Johnson, R. G. Johnson and L. M. Langer, Phys. Rev. **98** (1955) 1517
29) K. W. Ford, Phys. Rev. **98** (1955) 1516
30) S. Thulin, J. Mareau and H. Atterling, Ark. f. Fysik **8** (1954) 219
31) W. E. Graves and A. C. G. Mitchell, Phys. Rev. **97** (1955) 1033
32) S. Thulin, Ark. f. Fysik **9** (1955) 137
33) J. M. Hollander, I. Perlman and G. T. Seaborg, Revs. Modern Phys. **25** (1953) 469
34) G. Scharff-Goldhaber and M. McKeown, Phys. Rev. **92** (1953) 356
35) A. W. Schardt and J. P. Welker, Phys. Rev. **99** (1955) 810
36) D. R. Wiles, Phys. Rev. **93** (1954) 181
37) L. Grodzins and H. Motz, Bull. Amer. phys. Soc. **30** (1955) No. 7, 9
38) R. M. Steffen and J. N. Brazos, Phys. Rev. **99** (1955) 1646
39) J. N. Brazos and R. M. Steffen, Phys. Rev. **99** (1955) 1645
40) G. M. Temmer and N. P. Heydenburg, Phys. Rev. **98** (1954) 1308
41) H. Mark, C. McClelland and C. Goodman, Phys. Rev. **98** (1955) 1245
42) H. B. Mathur and E. K. Hyde, Phys. Rev. **95** (1954) 708
43) H. B. Mathur, Phys. Rev. **97** (1955) 707
44) E. Hebb, Phys. Rev. **97** (1955) 987
45) A. C. G. Mitchell and E. Hebb, Phys. Rev. **95** (1954) 727
46) C. H. Pruett and R. G. Wilkinson, Phys. Rev. **96** (1954) 1340
47) J. T. Jones and E. N. Jensen, Phys. Rev. **97** (1955) 1031
48) J. W. Mihelich and A. de Shalit, Phys. Rev. **93** (1954) 135
49) E. F. Kultz, W. W. Kuntz and W. G. Hartman, Uspehi Fizičeskih Nauk **55** (1955) 535
50) A. de Shalit, Helv. Phys. Acta **25** (1952) 279
51) M. G. Mayer and J. H. D. Jensen, Elementary theory of Nuclear Shell Structure (John Wiley & Sons, New York, 1955)
52) I. Bergström, private communication (1955)
53) M. Schmorak, private communication (1955)
54) A. H. Wapstra, D. Maeder, G. J. Nijgh and L. Th. M. Orenstein, Physica **20** (1954) 169
55) J. Varma, Phys. Rev. **94** (1954) 1688
56) J. R. Prescott, Proc. Phys. Soc. A **67** (1954) 254
57) V. E. Krohn and S. Raboy, Phys. Rev. (1955) **97** 1017
58) A. de Shalit, Phys. Rev. **90** (1953) 83
59) I. Bergström and A. H. Wapstra, Phil. Mag. **46** (1955) 61
60) I. Bergström, R. D. Hill and G. de Pasquali, Phys. Rev. **92** (1953) 918
61) R. W. Hayward and D. D. Hoppes, Phys. Rev. **101** (1956) 93
62) L. C. Aamodt and P. C. Fletcher, Phys. Rev. **98** (1955) 1224
63) E. Rasmussen and V. Middelboe, Z. Physik **141** (1955) 160
64) P. A. Axel, J. D. Fox and R. H. Parker, Phys. Rev. **97** (1955) 975

65) W. E. Meyerhof, L. G. Mann and H. I. West, Phys. Rev. **92** (1953) 758
66) J. P. Hobson, J. C. Hubbs and W. A. Nierenberg, Phys. Rev. **96** (1954) 1450
67) M. L. Perlman and J. P. Welker, Phys. Rev. **100** (1955) 81
68) B. Senitzky, I. I. Rabi and M. L. Perl, Phys. Rev. **98** (1955) 1537
69) R. W. Hayward, D. D. Hoppes and H. Ernst, Phys. Rev. **98** (1955) 231
70) K. Murakawa, Phys. Rev. **98** (1955) 1285
71) C. Levi and L. Papineau, Compt Rend. **238** (1954) 2313
72) K. G. Kessler and R. E. Trees, Phys. Rev. **92** (1953) 303
73) P. B. Sogo and C. D. Jeffries, Phys. Rev. **98** (1955) 1316
74) C. L. McGinnis, Phys. Rev. **97** (1955) 93
75) K. Althoff, Z. Physik **141** (1955) 33
76) G. Lührs, Z. Physik **141** (1955) 486
77) L. P. Gillon, K. Gopalakrishnan, A. de Shalit and J. W. Mihelich, Phys. Rev. **93** (1954) 124
78) J. B. Reynolds, R. L. Christensen, D. R. Hamilton, A. Lemonick, F. H. Pipkin and H. H. Stroke, Phys. Rev. **99** (1955) 613
79) King, Dismuke, Way, Oak Ridge National Laboratory, Report ORNL-1450 (unpublished, quoted in reference[21]))
80) J. W. Mihelich and M. Goldhaber, Phys. Rev. **98** (1955) 1185
81) F. Bitter, S. P. Davis, B. Richter and J. E. R. Young, Phys. Rev. **96** (1954) 1531
82) K. E. Bergkvist, I. Bergström, C. J. Herrlander, S. Hultberg, H. Slätis, E. Sokolowski, A. H. Wapstra and T. Wiedling, Phil. Mag. **46** (1955) 65
83) B. Saraf, Phys. Rev. **97** (1955) 715
84) C. L. McGinnis, Phys. Rev. **98** (1955) 1172
85) M. J. Glaubman, Phys. Rev. **98** (1955) 645
86) M. L. Perlman and J. P. Welker, Phys. Rev. **95** (1954) 133
87) N. Benczer, B. Farrelly, L. Koerts and C. S. Wu, Bull. Amer. phys. Soc. **30** (1955) 9
88) R. M. Sinclair, Bull. Amer. phys. Soc. **1** (1956) 42
89) G. Scharff-Goldhaber, Phys. Rev. **90** (1953) 587
90) R. S. Caird and A. C. G. Mitchell, Phys. Rev. **94** (1954) 412
91) H. L. Finston and W. Bernstein, Phys. Rev. **96** (1954) 71
92) M. McKeown and S. Katcoff, Phys. Rev. **94** (1954) 965
93) G. L. Keister, E. B. Lee and F. H. Schmidt, Phys. Rev. **97** (1955) 451
94) J. L. Olsen and G. D. O'Kelly, Phys. Rev. **95** (1954) 1539
95) J. W. Mihelich, M. McKeown and M. Goldhaber, Phys. Rev. **96** (1954) 1450
96) P. H. Stelson and F. K. McGowan, Phys. Rev. **99** (1955) 616
97) J. B. Swan, W. M. Portnoy and R. D. Hill, Phys. Rev. **90** (1953) 257
98) V. K. Fischer, Phys. Rev. **99** (1955) 672
99) D. E. Alburger and A. W. Sunyar, Phys. Rev. **99** (1955) 695
100) C. E. Mandeville, J. Varma and B. Saraf, Phys. Rev. **98** (1955) 94
101) C. McClelland, H. Mark and C. Goodman, Phys. Rev. **97** (1955) 1191
102) R. Thieberger and I. Talmi, Phys. Rev. **102** (1956) 923
S. Goldstein and I. Talmi, Phys. Rev. **102** (1956) 589
I. Talmi and R. Thieberger, Phys. Rev. (in press)

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INVESTIGATION OF COULOMB EXCITATION OF NUCLEI BY NITROGEN IONS

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Abstract: The ratio of the cross sections for Coulomb nuclear excitation by heavy particles and protons has been computed. It is shown that in many cases it is more profitable to use heavy ions than protons for Coulomb excitation of nuclei. Preliminary results of a study of Coulomb excitation of a number of nucleides with 15.6 MeV nitrogen ions are presented.

1. Theory

The cross section for Coulomb excitation of nuclei, in the case of quadrupole excitation, can be expressed by the formula

$$\sigma = \frac{m^2 v_f^2}{Z_2^2 e^2 \hbar^2} B_e(2) f_2(\xi) \tag{1}$$

where

$$\xi = \frac{Z_1 Z_2 e^2}{\hbar} \left(\frac{1}{v_f} - \frac{1}{v_i} \right). \tag{2}$$

Here m is the reduced mass of the bombarding particle, Z_1 the number of protons in its nucleus, v_i, v_f the initial and final relative velocities of the particle, Z_2 the number of protons in the nucleus of the target atom, $B_e(2)$ the reduced probability of electric quadrupole transition of this nucleus from the ground state to a given excited state; $f_2(\xi)$ is the Coulomb excitation function [1,2].

In many cases of excitation of nuclear levels the cross section σ has been found to be very small. In such cases it is usual to carry out experimental study of Coulomb excitation by using very energetic bombarding particles as the cross section σ sharply increases with the energy. However, the condition that these particles should not induce nuclear reactions in the target imposes a restriction on the upper energy of the particles. Occurrence of nuclear reactions significantly complicates the interpretation of the experimental results as well as the calculation of the Coulomb

excitation cross section. Thus, some of the γ-lines emitted by the target during irradiation may well be due to nuclear reactions instead of Coulomb excitation, while in other cases only part of the γ-quanta of a given line may result from Coulomb excitation.

The calculations presented below show that if heavy particles (and in particular nitrogen ions) with sufficiently high energy are used instead of protons for Coulomb excitation, large cross sections may be attained; on the other hand for such particle energies the probability of occurrence of nuclear reactions will be practically zero. Another advantage of utilizing heavy ions is that the interfering background of characteristic X-rays from the target should be comparatively much smaller than that encountered in proton work [3,4]).

We now compute the ratio σ_h/σ_p, where σ_h is the cross section for Coulomb excitation of a nuclear state by heavy ions and σ_p is the cross section for Coulomb excitation of the same state by protons (of different energy, see below).

The dimensionless parameter ξ_p can be written as follows:

$$\xi_p = 0.1575 Z_1 Z_2 \sqrt{\mu_p}\left(\frac{1}{\sqrt{E_p - \Delta E}} - \frac{1}{\sqrt{E_p}}\right), \qquad (3)$$

where μ_p is the reduced proton mass in nuclear mass units, E_p is the collision energy for protons bombarding the given target, ΔE is the excitation energy of the given nuclear state (in MeV). ξ_h can be calculated from an analogous formula.

It follows from (1)

$$\frac{\sigma_h}{\sigma_p} = \frac{\mu_h}{\mu_p} \cdot \frac{E_h - \Delta E}{E_p - \Delta E} \cdot \frac{f_2(\xi_h)}{f_2(\xi_p)}. \qquad (4)$$

The values of the function $f_2(\xi)$ necessary for the calculations have been taken from ref. [5]). It is assumed that Coulomb excitation is induced by ${}_7N^{14}$ nitrogen ions and that $E_h = 7E_p$. The curves $\sigma_h/\sigma_p = F(\xi_p)$ for three targets with significantly different values of Z_2 are shown in fig. 1. In each case the parameter of the family of curves is $k = \Delta E/E_p$. For any given value of k a definite energy E_p corresponds to each value of ξ_p. Some of the values of E_p are presented in fig. 1.

As can be seen from the graphs, the ratio σ_h/σ_p is much greater than unity for ξ_p lying between 0 and 1 (which includes all cases of practical interest). For a given value of k, σ_h/σ_p is maximum when $\xi_p \to 0$, that is, at large values of E_p.

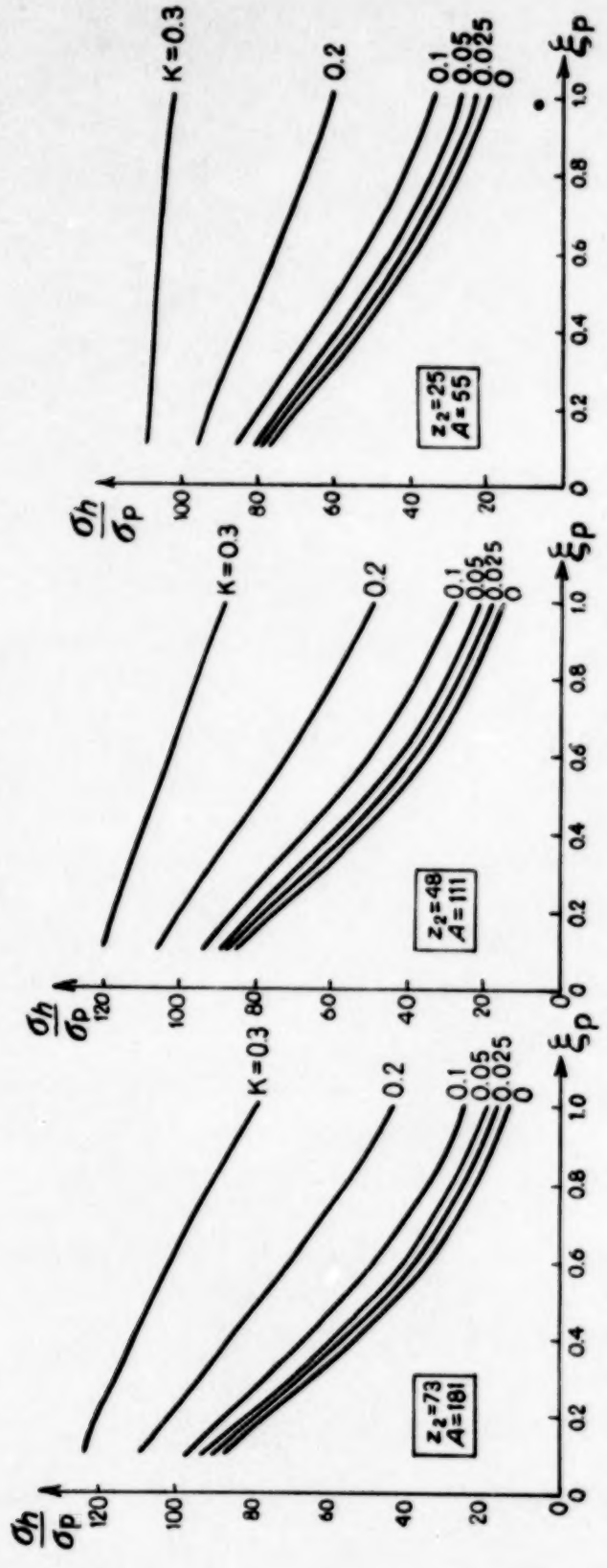

Fig. 1. Dependence of the ratio σ_h/σ_p on ξ_p for different values of $k = \Delta E/E_p$. Curve "0" is computed from formulae of semiclassical theory [1, 6] which are valid for $k \to 0$. The tables on p. 68 give the values of E_p (in MeV) in terms of ξ_p and k.

k \ ξ_p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.3			56	31	20	14	10.2	8	6.2	5
0.2		46	20	11	7.3	5.1	3.7	2.9	2.3	1.8
0.1	38	9.6	4.3	2.4	1.5	1.1	0.8	0.6	0.48	0.38
0.05	8.8	2.2	0.98	0.55	0.35	0.24	0.18	0.14	0.11	0.09
0.025	2.14	0.54	0.24	0.13	0.09	0.06	0.04	0.033	0.026	0.021

k \ ξ_p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.08	0.9	1.0
0.3		54	24	13.5	8.6	6	4.4	3.4	2.7	2.2
0.2		20	8.8	4.9	3.2	2.2	1.6	1.2	0.98	0.79
0.1	16.6	4.2	1.8	1	0.66	0.46	0.34	0.26	0.2	0.17
0.05	3.8	0.96	0.43	0.24	0.15	0.11	0.08	0.06	0.05	0.04
0.025	0.92	0.23	0.10	0.06	0.04	0.026	0.019	0.014	0.011	0.009

k \ ξ_p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.3	58	14.5	6.4	3.6	2.3	1.6	1.2	0.9	0.7	0.6
0.2	21	5.3	2.4	1.3	0.9	0.6	0.4	0.33	0.26	0.21
0.1	4.5	1.1	0.5	0.28	0.18	0.12	0.09	0.07	0.06	0.04
0.05	1.03		0.114		0.041		0.021		0.013	
0.025	0.25		0.028		0.010		0.005		0.003	

If Coulomb excitation of the first and second excited states in tantalum ($\Delta E_1 = 0.137$ MeV; $\Delta E_2 = 0.303$ MeV) are being investigated and the target is bombarded with 15.6 MeV nitrogen ions (i.e., $E_h = 14.5$ MeV and $E_p \approx 2.1$ MeV) then, as calculations by formula (4) show, $(\sigma_h/\sigma_p)_1 \approx 75$ and $(\sigma_h/\sigma_p)_2 \approx 50$.

If some value E'_h is prescribed one can determine the proton energy for which σ_p will be equal to $\sigma_h(E'_h)$. Plotting in accordance with formula (1) graphs of $\sigma_p(E_p)$ for values of E_p up to 7 MeV and comparing the data of these graphs with the value of σ_h/σ_p for nitrogen ions at $E'_h = 14.5$ MeV and various values of ΔE it will be found that, for nuclei which are similar to $_{73}Ta^{181}$ with respect to A and Z on excitation of the level with $\Delta E = 0.5$ MeV the equality $\sigma_p = \sigma_h(14.5\text{ MeV})$ will hold if $E_p = 5.3$ MeV (at lower proton energies $\sigma_p < \sigma_h(14.5\text{ MeV})$); similarly, if $\Delta E = 0.2$ MeV then $\sigma_p = \sigma_h$ only for $E_p \gg 7$ MeV. For nucleides of the $_{48}Cd^{111}$ type and $\Delta E = 0.6$ MeV one gets $\sigma_p = \sigma_h$ if $E_p = 5.7$ MeV; if $\Delta E = 0.5$ MeV then $\sigma_p = \sigma_h$ for $E_p \gg 7$ MeV. Finally, for nucleides of the $_{25}Mn^{55}$ type at all values $\Delta E < 0.7$ MeV one obtains $\sigma_h > \sigma_p$ if $E_p < 7$ MeV.

In order to increase the γ-quantum yield experimental studies of Coulomb excitation are usually carried out with thick targets. The ratio of the yields of γ-quanta following Coulomb excitation of a certain state is given by the formula

$$\frac{Q_h}{Q_p} = \frac{\int_{E_h}^{E_h^*} \sigma_h(E) \dfrac{dE}{(dE/dx)_h}}{\int_{E_p}^{E_p^*} \sigma_p(E) \dfrac{dE}{(dE/dx)_p}}, \qquad (5)$$

where E^* is defined by the condition $\sigma(E^*) \approx 0$.

It is implied in this formula that an equal number of protons and heavy ions are incident on the target per second.

For excitation of the first nuclear level in Ta^{181} by nitrogen ions with $E_h = 14.5$ MeV formula (5) yields $Q_h/Q_p = 12$. The proton energy loss in tantalum, $(dE/dx)_p$, is involved in the calculations and its value was obtained by interpolating the data presented in ref.[7,8]). In the proton energy range from 2.1 to 0.8 MeV this quantity is sufficiently constant and is equal to 0.0645 MeV cm^2/mg. For nitrogen ions slowed down in tantalum $(dE/dx)_h$ was evaluated by recalculating the α-particle range-energy curve in tantalum by the method proposed by Longchamp[9]). In the

nitrogen ion energy range between 15.6 and 8 MeV it is sufficiently constant and equals 2.32 MeV · cm²/mg.

Thus, in the case under consideration, when a thick target is used the ration Q_h/Q_p remains larger than 1 even though it is considerably smaller than σ_h/σ_p (12 instead of 75).

The cross section for production of X-ray K-quanta as a result of ionization of the target atoms by relatively slow charged particles[10]) is approximately equal to $Z_1^2(E_1/A_1)^4$. By applying this relation and the data presented above it can be computed that in the case of a thick tantalum target the ratio of the number of X-ray K-quanta, due to ionization of the Ta atoms by the incident particles, to the numbers of 137 keV γ-quanta arising from 15.6 MeV nitrogen ions is 15 times smaller than when 2.1 MeV protons are used.

Experiments carried out by us confirm these conclusions. We investigated Coulomb excitation by nitrogen ions in 21 elements. The experimental method was chosen with the aim of studying two independent problems: 1) determination of the energy of the lower nuclear excited levels and 2) calculation of the corresponding reduced transition probabilities, $B_e(2)$. With a knowledge of $B_e(2)$ it is also possible to calculate the intrinsic quadrupole moment of a given nucleus if it is known that the excited level under consideration is a rotational level.

The results given in the present paper refer only to the first of the problems mentioned above.

2. The Experimental Method

Triply charged 15.6 MeV nitrogen ions were obtained from the cyclotron of the Leningrad Physico-Technical Institute of the Academy of Sciences, USSR. The ion source had a graphite anode and thick tantalum wire cathode (ø 3.5 mm). The "extraction electrode" which was mounted on one of the dees was located at a short distance from the anode slit. A separate direct current generator was used for the cathode heating.

The work was performed with a beam extracted by means of a conventional deflector; only a small part of the extracted beam which passed through an oval shaped opening 20 mm high and 10 mm wide was utilized (the plate with the aperture was located in plane a, fig. 2). Thus the triply charged ion beam travelling towards the target was sufficiently mono-energetic. Only those ions were present in the triply charged particle beam which left the ion source

with a triple charge and were subsequently accelerated in the cyclotron by the fundamental frequency of the accelerating field. The probability of singly charged ions leaving the ion source to traverse the whole deflector channel after losing two additional electrons in collisions with molecules of the residual gas is very small as only in exceptional cases the centre of curvature of the last circle on the orbit can be close to the geometrical centre of the chamber.

It was found in the experiments that in the deflected beam singly charged nitrogen ions are present simultaneously with triply charged ions. These ions were emitted from the ion source with a single charge, were then accelerated in the cyclotron by the third harmonic of the accelerating field and, without changing their charge during acceleration, passed through the deflector channel. Obviously, the magnetic rigidity of such ions is the same as that of triply charged ions and their energy is therefore 9 times smaller.

In order to get experimental data which could be used to compute the *absolute* value of the Coulomb excitation cross section it was necessary to remove the singly charged ions from the beam. The energy of these ions is too low to create Coulomb excitation γ-rays but the ion current on the target would be the sum of singly and triply charged ions of an unknown ratio.

The singly and triply charged ions were separated by deflection in an electric field. The beam was sent between the horizontal plates of a plane condenser supplied with a voltage of ≈ 14 kV ($E \approx 7$ kV/cm). The length of the condenser was ≈ 25 cm.

After the condenser the triply charged ion beam passed through a hole ø 30 mm located in plane b (fig. 2) and after that through a system consisting of two magnetic quadrupole lenses which focused it on the target.

The main parameters of each of the lenses were as follows: aperture diameter 4 cm, radius of curvature of the pole cylindrical surface 2.3 cm, lens length 10 cm, lens weight (with coils) 50 kg, total number of turns in the four coils ≈ 4000, current at optimal focusing ≈ 1.8 A. † The distance of the median plane of the first lens from the exit window of the cyclotron chamber was ≈ 1.6 m; the median lens planes were 50 cm apart and the target was at a distance of 1.25 m from the median plane of the second lens.

† The current in the lens coils which ensured optimal focusing of the beam depending on the position of the focusing plane was established in special experiments.

Inclusion of the magnetic lens in the arrangement led to an increase of intensity of the nitrogen ion beam incident on the target by about 5 times and made it possible to remove the detecting apparatus at a considerable distance from the cyclotron.

The target, which was a disk 28 mm in diameter, was placed at the bottom of an isolated metallic vessel which served as a Faraday cage. A slide-block with a ZnS disk deposited on it was arranged in front of the cylinder. With the help of a small solenoid and an iron core connected to the slide block the latter could be moved in the vacuum and open the entrance to the Faraday cage or place the ZnS layer in the path of the beam. The glowing spot due to the beam striking the ZnS layer could be observed through the walls of the glass tube. This device was used to check the centering of the beam and to control the focusing. At optimal currents in the lens coils the dimensions of the spot on the screen were ≈ 5 mm in height and ≈ 14 mm in width.

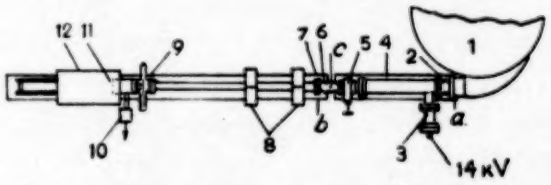

Fig. 2. Diagram of experimental arrangement.
1 – cyclotron chamber, 2 and 6 – sylphons, 3 – porcelain leading-in insulator for voltage applied to one of plates of the deflecting condenser, 4 and 7 – rods connected by gimbal joint, 5 – vacuum valve closed when target is changed, 8 – magnetic lenses, 9 – glass for observation of glow of the screen, 10 – input stage of current integrator, 11 – target, 12 – iron housing with photomultiplier.

Rods 4 and 7 (fig. 2) were connected by means of a cardan joint which permitted rod 7 to be turned about point c in any direction; thus, the system could easily be arranged to ensure that the central part of the target was struck by the beam.

The amount of electricity in the beam striking the target was measured with a vacuum tube current integrator which was calibrated before and after each measurement of the Coulomb excitation γ-spectrum (for this purpose a known current from a separate source was passed through the integrator). In order to exclude possible errors in measurement of the beam current due to secondary electron emission a ring at -90 V potential with respect to the Faraday cage was placed before the latter. The triply charged nitrogen ion current at the target reached 0.5 μA (the total current

of the deflected beam measured in plane a, fig. 2, comprised 10—15 μA).

The γ-radiation produced during irradiation of the target was measured with a scintillation γ-spectrometer employing a NaJ(Tl) crystal. The crystal dimensions were ø 29 mm, height 13 mm. A block-diagram of the spectrometer is shown in fig. 3. The photo-

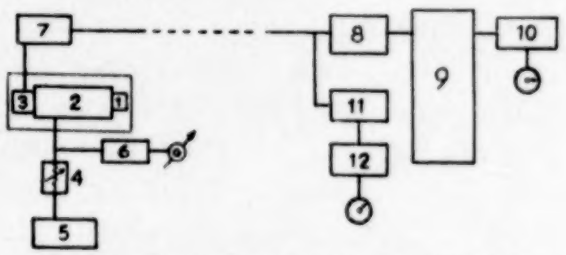

Fig. 3. Block-diagram of γ-spectrometer.
1 – scintillator, 2 – photomultiplier, 3 – cathode follower, 4 – adjustment resistor, 5 – dry battery, 6 – potentiometric circuit for voltage measurement, 7 – powerful cathode follower, 8 – linear amplifier, 9 – 50-channel pulse-height analyzer, 10 and 12 – scalers with scaling factors 1024, 11 – integral discriminator.

multiplier tube, ФЭУ-С type, and the first cathode follower were placed inside the reliable antimagnet shielding box consisting of three concentric iron cylinders. A dry battery was used to feed the multiplier tube ($V \approx 1.2$ kV). The voltage supplied to it could be regulated by means of a potentiometer circuit and was kept constant with an accuracy to within 0.01 %. A powerful cathode follower fed the multiplier pulses through a ≈ 50 m high frequency cable to a linear amplifier with negative feedback after which the pulses were fed to a 50-channel pulse-height analyzer.† The principle of transformation of amplitude into duration is utilized in this apparatus. An interlocking stage was included in the analyzer; this disconnected the analyzer input "9" from the amplifier output "8" during 16 m sec. after arrival of each pulse at the analyzer. This is just the time required for the pulse-height analysis to be carried out, after which a numerator in one of the channels is triggered. Under our experimental conditions the number of pulses fed to the input of amplifier "8" during an experiment, N_0, considerably exceeded the number of pulses N_B which were subjected to pulse-height analysis in analyzer "9". Evidently, in order to evaluate the absolute magnitudes of σ from the experimental spectrogram it is necessary to know N_B/N_0. A fast integral

† The analyzer was constructed by L. N. Galperin.

discriminator "11" and scaler "12" were therefore added to the experimental arrangement; with their aid N_0 could be measured. The total amount of times the interlocking stage was tripped during the same period (i.e., N_B) was measured with scaler, "10".

An adjustable bias voltage, V_b, could be included in the analyzer circuit; the height of a pulse fed to the measuring circuit was $V_a - V_b$, where V_a is the amplitude of the initial pulse. In each experiment the discrimination level in discriminator "11" was adjusted so as to equal the lower discrimination level of analyzer "9".

A pulse generator (calibrator) fed to block "9" and a precision voltmeter were used to carry out the voltage calibration of the pulse-height analyzer in each experiment. In all the measurements the analyzer channel width was 1 V.

Calibration of the γ-spectrometer as a whole was carried out by the γ-lines of radioactive sources: Ce^{141} (146 keV), Hg^{203} (279.5 keV), Cs^{137} (661 keV) and Co^{60} (1.17 and 1.33 MeV). The choice of any of these sources in an experiment was made in accordance with the condition that the energy of the calibration γ-line should be as close as possible to that of the investigated γ-line.

To illustrate the resolving power of the spectrometer it may be mentioned that the relative half-width of the Cs^{137} γ-line photo-peak was 9.5 %.

It should be mentioned that in our arrangement the following material which could partly absorb the target γ-radiation was located between the target and scintillator: 0.19 mm Cu, 1 mm Al, 1 mm MgO. In some cases the target was backed by copper from 80 μ to 0.5 mm thick which was also a γ-ray absorber.

3. Results

Our data on the excitation energies for a number of nucleides are presented in table I. When known, the mass number of the nucleus which the given level belongs to is also cited.

No γ-lines were observed on bombardment of K, Ni, Cu, Sn and Bi. With the exception of K metallic targets were irradiated and the results of these experiments were applied to determine the corrections for bremsstrahlung which arises when the nitrogen ions are slowed down in the various targets (these corrections are necessary in order to compute the absolute values of the cross section σ).

Some of our spectrograms are reproduced in fig. 4 to 11. The background from cosmic rays and bremsstrahlung is subtracted.

The number of γ-quanta recorded by the scintillation counter pertaining to a microcoulomb of nitrogen ions and a γ-quantum energy interval of 10 keV is plotted along the ordinate axis.

TABLE I

Irradiated nuclei and γ-quantum energy in keV							
F^{19}_{9}	125 205	Ge^{73}_{32}	72	Ag_{47}	310 409	Ta^{181}_{73}	138 301
Na^{23}_{11}	435	Se^{77}_{34}	238 452	Cd^{113}_{48} Cd^{111} Cd^{114}	297 325 543	W_{74}	112
V^{51}_{23}	320	Mo^{95}_{42} Mo^{100}	210 540	In^{115}_{49}	562	Au^{197}_{79}	286 580
Mn^{55}_{25}	127 590	Rb^{103}_{45}	295 358	J^{127}_{53}	60 205	Pb^{207}_{82}	580

The γ-quantum energy was determined with an accuracy of $\approx$ 1 to 3 %.

Some of the spectrograms were obtained when a bias voltage was used in the pulse-height analyzer (fig. 5b, 7, 9b, 10, 11). In this case the analyzer was not loaded or blocked by a large number of small pulses from low energy γ-quanta; thus, the number of counts needed to measure the more energetic γ-quanta could be recorded in a considerably shorter time.

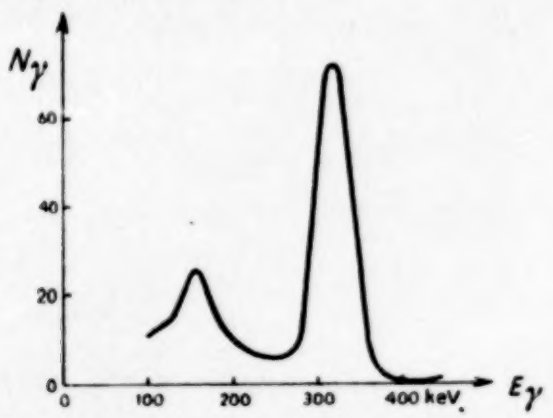

Fig. 4. Gamma spectrum for Coulomb excitation of vanadium.

In most cases the γ-lines observed by us were also observed in other investigations in which protons or α-particles were used for Coulomb excitation of the nuclei.

A $\approx$ 560 keV γ-line (fig. 7) was observed on bombardment of indium with nitrogen ions; we did not find the $\approx$ 500 keV line

reported in ref. [11, 12]). It should be noted that data on the β-disintegration of Cd^{115} indicate the existence of an excited level of 595 keV energy [13]) in In^{115}.

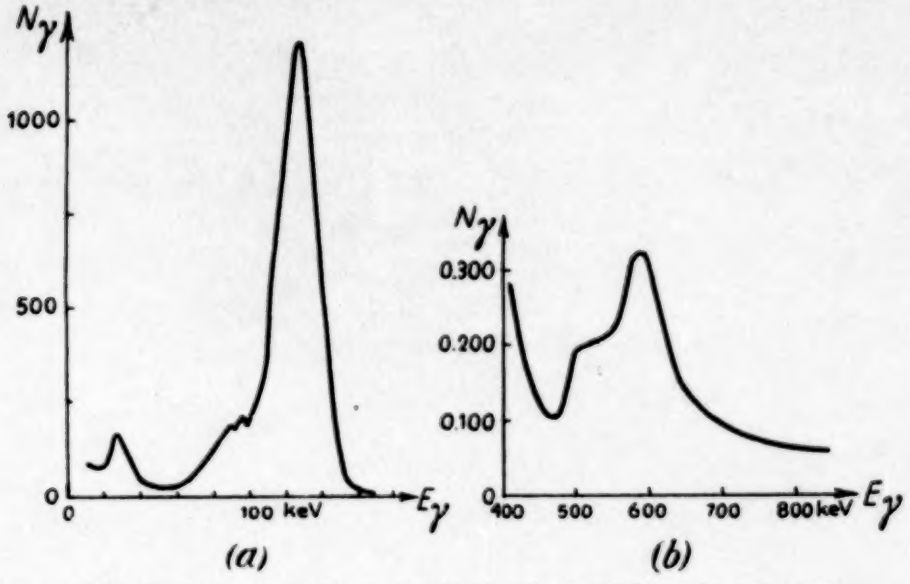

Fig. 5. Gamma spectrum for Coulomb excitation of manganese.

The Coulomb excitation γ-spectrum of tantalum is shown in fig. 9. Peak A is due to the X-ray K radiation of $\approx$ 57 keV energy from tantalum. On basis of the graphs presented in ref. [14]) one can compute $(I_K/I_\gamma)_p$, the ratio of the number of X-ray K-quanta produced by 2.1 MeV protons in tantalum to the number of 137 keV

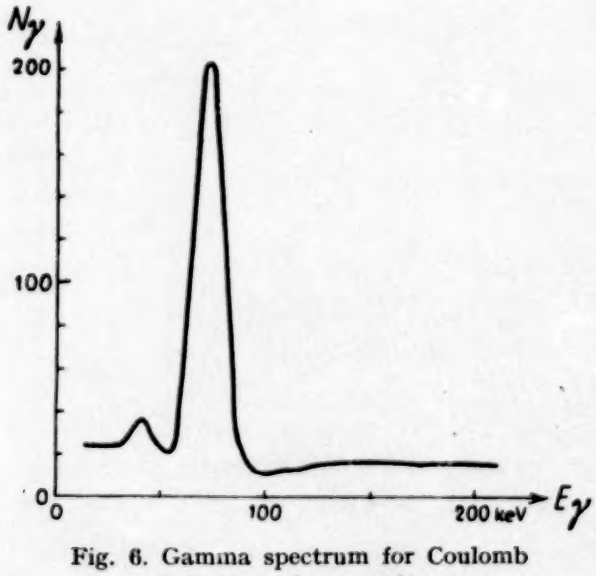

Fig. 6. Gamma spectrum for Coulomb excitation of germanium.

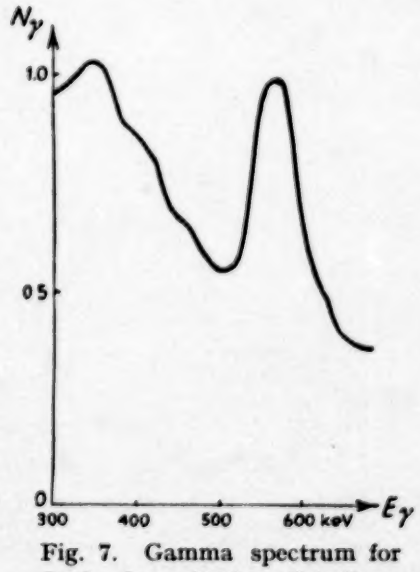

Fig. 7. Gamma spectrum for Coulomb excitation of indium.

γ-quanta due to Coulomb excitation of tantalum. This ratio was found to equal 12.4. In accord with the calculations performed in sec. I we should expect that in our experiments $(I_K/I_\gamma)_h = 12.4/15 \approx 0.83$. This is approximately what was obtained; in our experiments we found ≈ 0.65 (this value was obtained after subtracting

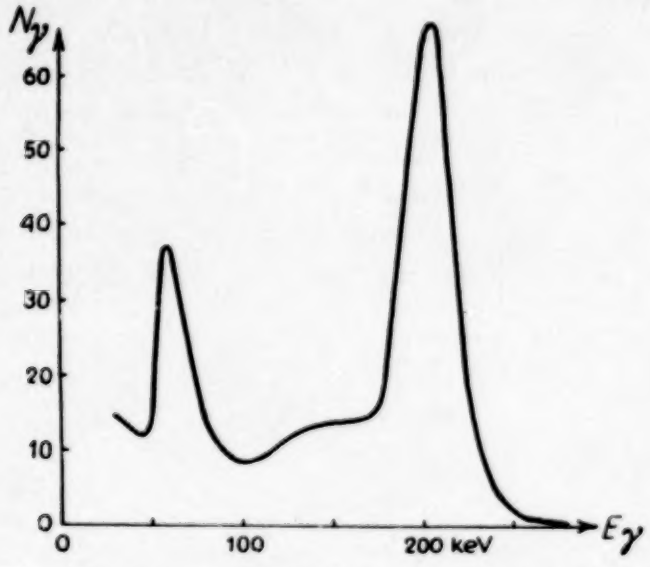

Fig. 8. Gamma spectrum for Coulomb excitation of iodine.

from the total number of K-quanta those quanta which are due to internal conversion in the K-shell during the transition with $\Delta E = 137$ keV). Our result confirms Henneberg's theory which underlies the calculation carried out in ref. [4]).

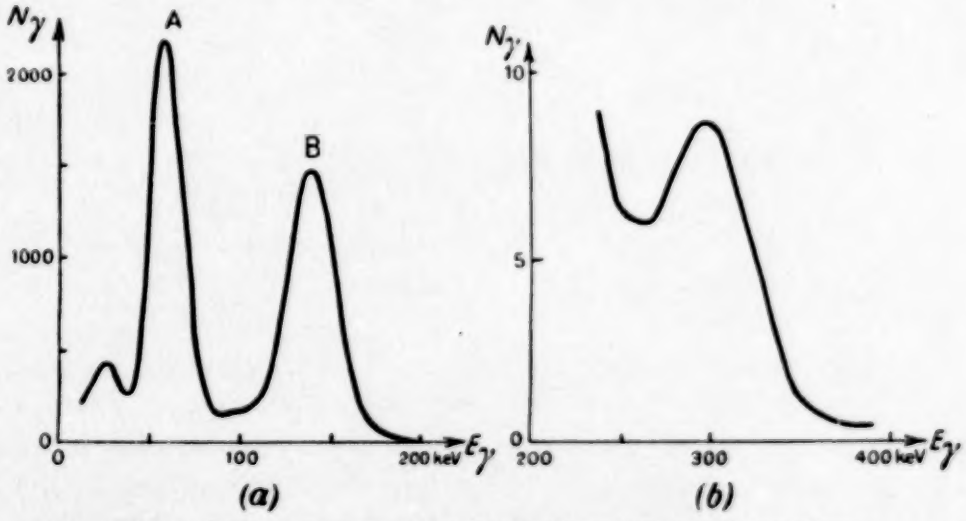

Fig. 9. Gamma spectrum for Coulomb excitation of tantalum.

The excitation probability for the second rotational level of Ta^{181} ($\Delta E = 303$ keV) is much smaller than that for the first level. The explanation of this is that the reduced probability of transition to the second level is four times smaller than that for transition to the first level and also that the function $f_2(\xi)$ rapidly decreases with increasing ξ. Moreover, the relative Coulomb excitation probability for nitrogen ions (compared to proton excitation) decreases with increasing energy of the excited level. Nevertheless, by introducing a suitable bias voltage in the pulse-height analyzer it is easy to observe 301 keV γ-rays (fig. 9b).

An inspection of our data shows that on bombardment with nitrogen ions only γ-lines resulting from Coulomb excitation can be detected in the γ-spectra of nuclei with $Z_2 \geqq 9$; no γ-lines

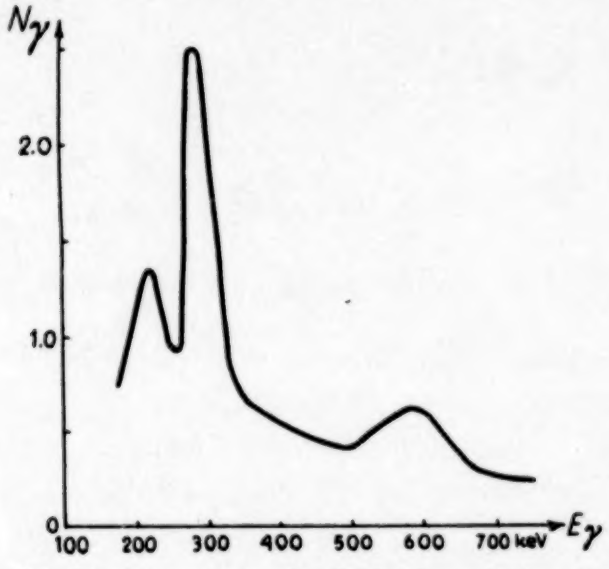

Fig. 10. Gamma spectrum for Coulomb excitation of gold.

attributable to nuclear reactions were observed. Thus, the 420, 650 and 975 keV Mn lines [12]), 440 keV Rh line [15]) or 420 and 500 keV Cd lines [12]) are not present in our spectrograms. This confirms the assumptions made in the above-mentioned papers that these γ-lines should be attributed to nuclear reactions and not to Coulomb excitation.

There is some evidence which seems to indicate that when a target is bombarded by heavy ions nuclear reactions can be induced not only via the formation of a compound nucleus which the incident particle completely enters but also by break-up to the particle into α-particles, protons and neutrons, one or several of

which penetrate the target nucleus [16]). In this latter case the reaction threshold will be considerably lower than that one would expect from the Coulomb barrier computed for the bombarding nucleus as a whole. It is possible that if higher energy nitrogen ions (compared to those in this investigation) are used for Coulomb excitation of nuclei this "buckshot effect" may be appreciable and then the advantage of working with heavy ions will be lost. A special experimental investigation is needed to determine the dependence of the permissible upper limit of the heavy particle energy on the atomic number of the target nucleus.

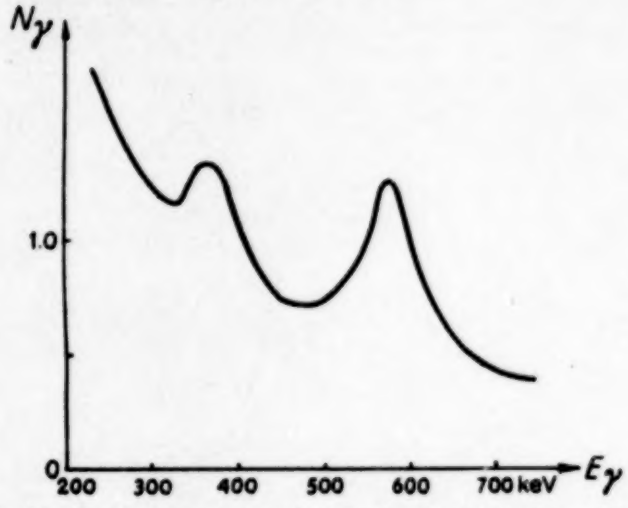

Fig. 11. Gamma spectrum for Coulomb excitation of lead.

In investigations of Coulomb excitation of nuclei by protons or α-particles difficulties due to small admixtures of light elements in the heavier elements under investigation were frequently encountered. The reason for this is that heavy elements were investigated with high energy particles as there was no danger of nuclear reactions occurring; however, nuclear reactions may occur in the nuclei of the admixture. Although no special precautions were taken to purify the target material we did not encounter any interfering effects which could be attributed to impurities.

The authors would like to thank L. N. Galperin, A. B. Girshin and S. P. Dmitriev for their helpful assistance during the above investigation.

References

1) K. A. Ter-Martirosyan, JETP **22** (1952) 284
2) K. Alder and A. Winther, Phys. Rev. **96** (1954) 237

3) D. L. Hill, Phys. Rev. **93** (1954) 923 (Kl)
4) A. P. Greenberg and I. Kh. Lemberg, JETP **30** (1953) (in press)
5) K. Alder and A. Winther, CERN/T/KA-AW-1 (Oct. 1954)
6) K. Alder and A. Winther, Phys. Rev. **91** (1953) 1578
7) H. A. Bethe and J. Ashkin, in Experimental nuclear physics, Ed. E. Segrè, Vol. I (1953) p. 166
8) B. V. Rybakov, JETP **28** (1955) 651
9) J. P. Longchamp, J. Phys. Radium **14** (1953) 89
10) W. Henneberg, Z. Phys. **86** (1933) 592
11) G. M. Temmer and N. P. Heydenburg, Phys. Rev. **93** (1954) 351
12) H. Mark, C. McClelland and C. Goodman, Phys. Rev. **98** (1955) 1245
13) J. Varma and C. E. Mandeville, Phys. Rev. **97** (1955) 977
14) T. Huus and Č. Zupančič. Dan. Mat-Fys. Medd. **28** (1953) No. 1
15) P. H. Stelson and F. K. McGowan, Phys. Rev. **99** (1955) 112
16) K. F. Chackett, J. H. Fremlin and D. Walker. Phil. Mag. **45** (1954) 173

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THE FORMATION OF He^5 BY THE DEUTERON BOMBARDMENT OF Li^7

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Abstract: The reaction Li^7(d, α) He^5 (n) He^4 has been studied at a deuteron energy of 900 keV by observing coincidences between alpha-particles concurrent with a measurement of their energies. The results are compared under certain assumptions and are consistent with the properties of He^5 as given by the scattering of neutrons in He^4. It is estimated that less than one tenth of the disintegrations observed are due to the alternative reactions Li^7(d,n) Be^9(α) He^4 and Li^7(d,nα)He^4.

1. Introduction

The deuteron bombardment of Li^7 results in the production of two alpha-particles and a neutron with a total energy release of 15.11 MeV. The reaction can be considered to proceed via a compound nucleus state in Be^9 which then disintegrates by one of the following processes:

$$Be^{9*} \rightarrow He^4 + He^5\ ,\ He^5 \rightarrow n + He^4 \qquad \text{(a)}$$

$$Be^{9*} \rightarrow n + Be^8\ ,\ Be^8 \rightarrow He^4 + He^4 \qquad \text{(b)}$$

$$Be^{9*} \rightarrow n + He^4 + He^4. \qquad \text{(c)}$$

It is possible to distinguish (a) and (b) by means of groups present in the spectrum of particle energies. A monoenergetic alpha-particle group has been observed [1]), which can only be due to a two body break-up of Be^{9*} leaving He^5 in the ground state. Two neutron groups have been observed [1]), corresponding to the formation of Be^8 in the ground and first excited states. No substantial evidence exists for reaction (c).

At high values of the alpha-particle energies reaction (a) is predominant and in the present experiment observations were made in this region by measuring the energy distribution of one of the alpha-particles for a particular energy of the other. This energy distribution depends on the relative probability for the formation of He^5 at given excitations and also on the angular correlation between the two alpha-particles.

The energy distributions observed in the present experiment are compared with calculated distributions based on the properties of He^5 as determined from neutron scattering in He^4 [2, 3]). The calculations require knowledge of the spin of the intermediate state of Be^9 which is obtained from the measured angular correlation and the known spin of the ground state of He^5.

2. Experimental Method

With the energy and direction of one alpha-particle fixed, however Be^{9*} breaks up, the second alpha-particle is emitted within a cone whose axis is collinear with the first. In the case of reaction (a) the semi-angle of this cone depends on the energy of break up of He^5 and has a maximum value of 25° in the present observations. The energy and direction of one of the alpha-particles was selected by a peaked proportional counter (counter I) which subtended a cone of semi-angle 2° at the target. The energy of the other alpha-particle was measured by a large multiplewire proportional counter (counter II) which subtended a cone of semi-angle 60° at the target. A diagram of the target chamber indicating the position of the counters is shown in fig. 1.

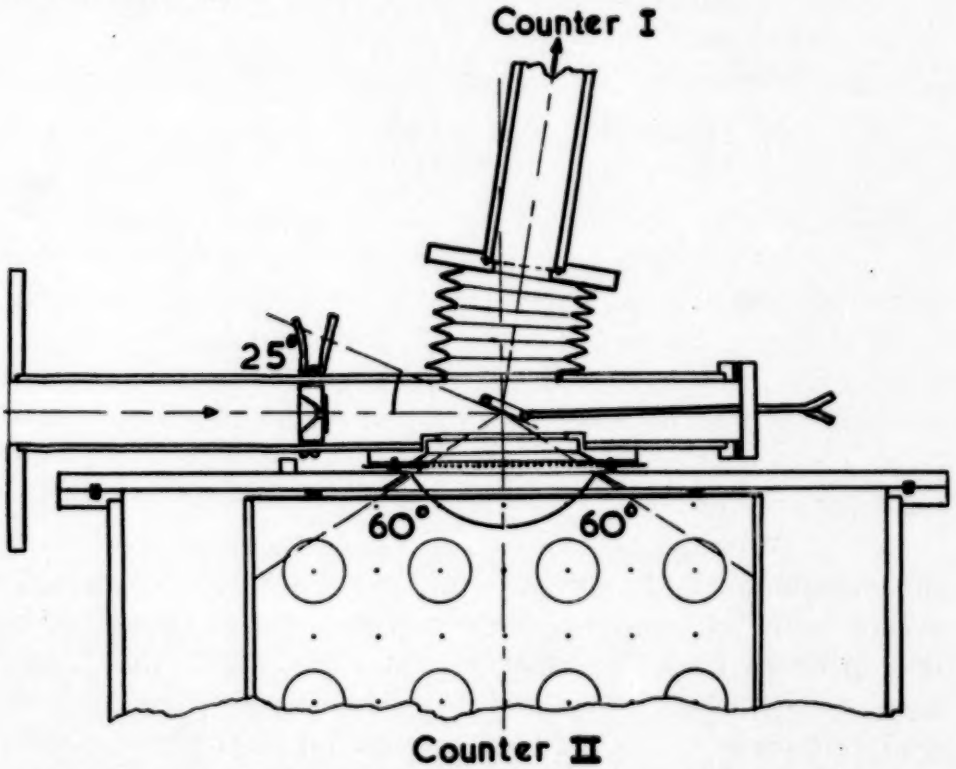

Fig. 1. Diagram of the target chamber showing the position of the counters

Alpha-particles detected in counter I passed through an air absorption cell with thin mica windows at each end and then entered a shallow proportional counter whose effective depth was equivalent to 2.5 mm of air. A discriminator at the output of the counter amplifier was adjusted to detect only those alpha-particles which ended their range in the counter; this range was varied by changing the pressure in the air absorption cell. The calibration was obtained by observing alpha-particle groups from the reactions $Li^7(p, \alpha)He^4$ and $Li^6(d, \alpha)He^4$. It was determined that at zero pressure the total air equivalent of counter and windows was 5.5 cm and that the air cell was 14.6 cm long. The natural width of the calibration groups was neglected and assuming the resolution of the counter to be Gaussian in shape, the standard deviation was found to be 0.18 MeV for 9 MeV alpha-particles.

Alpha-particles entered counter II, which has been described elsewhere [4]), through an aluminium window equivalent to 1.0 cm of air and passed through a space equivalent to 0.5 cm of air inside the counter before entering the sensitive volume. After amplification, pulses from the counter were recorded by a pulse height analyser when a coincidence occurred between the two counters within a total resolving time of 3.45 μs. To calibrate the analyser the alpha-particles from the reaction $Li^7(p, \alpha)He^4$ were observed in both counters in coincidence. This coincidence method of calibration ensured that all alpha-particles recorded by the analyser traversed the window at a definite angle and so suffered the same energy loss. The standard deviation for the group was 0.37 MeV.

A thin target backing was necessary to allow observation of alpha-particles emitted in opposite directions, hence targets were made by evaporating lithium aluminium hydride onto aluminium foils equivalent to 2 mm of air. The thickness of the deposit was measured by observing the width of the 440 keV (p, γ) resonance in Li^7 and was found to be equivalent to 1 mm of air. The plane of the foils was at 25° to the deuteron beam, at 65° to the centre axis of counter II and at 75° to the axis of counter I. The deuteron beam from the Canberra 1.2 MeV High Tension Set was analysed by magnetic deflection and collimated by an elliptical aperture so shaped that each counter saw an approximately circular area of target under bombardment.

3. Experimental Energy Distributions

The energy distribution of the alpha-particles produced by the deuteron bombardment of Li^7 has been measured previously by French and Treacy [5]) and others [1]), who found a monoenergetic group due to the formation of the ground state of He^5. This energy distribution has been measured by using counter I and varying the absorption cell pressure. The distribution observed for a deuteron energy of 900 keV is shown in fig. 2 where a smooth curve has been

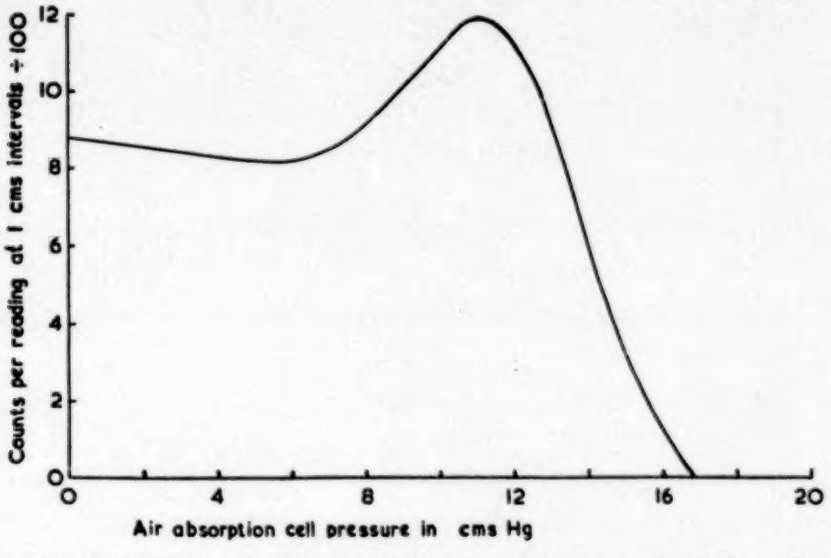

Fig. 2. The distribution of alpha-particles observed in counter I at a deuteron energy of 900 keV.

drawn through the experimental points. This distribution is similar to those also observed at deuteron energies of 300, 500 and 700 keV in this experiment and to those obtained in previous studies.

The alpha-particle (α_1) emitted from Be^{9*} and the alpha-particle (α_2) from the break-up of He^5 both contribute to the distribution in Figure 2. The group centred about 11.4 cm Hg is due to the emission of α_1 with an energy of 8.25 MeV leaving He^5 in the ground state. At 0 cm Hg the energy of the alpha-particles detected in counter I is 6.78 MeV and an α_1 emitted with this energy leaves He^5 at an excitation of 2.70 MeV above the ground state. For these reasons the energy distributions of the alpha-particles detected in counter II have been measured in coincidence with alpha-particles of 8.20 and 6.78 MeV energies in counter I. The distributions were measured at a deuteron energy of 900 keV and are shown in figures 3 and 4, respectively, where the experimental points have been corrected

for random coincidences and plotted with their probable errors. A random coincidence distribution was obtained by using artificial pulses in place of the output from counter I. The solid and dashed curves in figures 3 and 4 are calculated distributions which will be

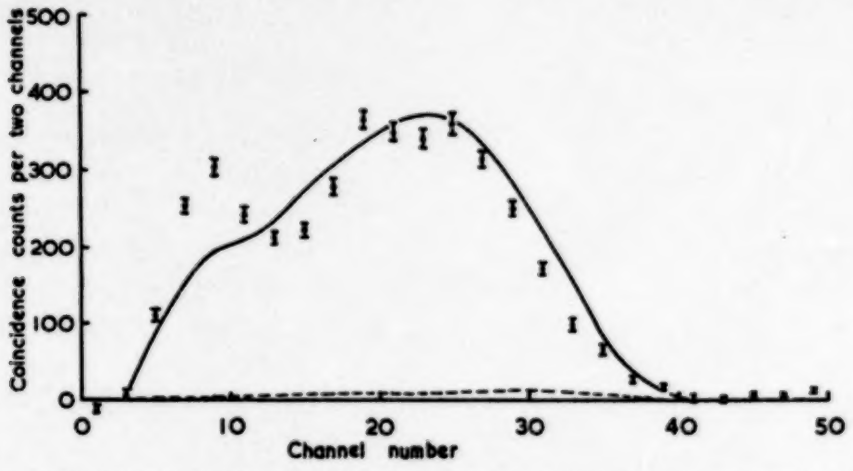

Fig. 3. Pulse height distribution of alpha-particles detected in counter II in coincidence with alpha-particles of energy 8.20 MeV detected in counter I.

described in the next section. Both spectra were recorded for the same number (to within 5 per cent) of disintegrations of Be^{9*} as determined by the total integrated beam current and also by the total number of particles detected in counter II.

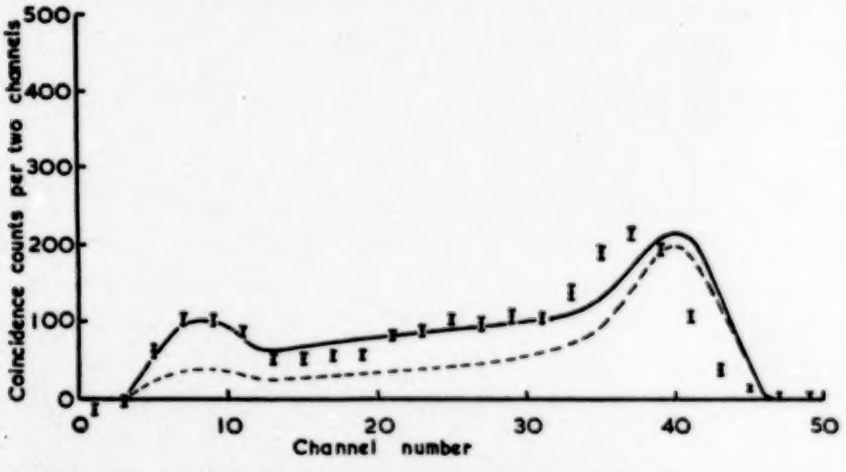

Fig. 4. Pulse height distribution of alpha-particles detected in counter II in coincidence with alpha-particles of energy 6.78 MeV detected in counter I.

A third energy distribution was recorded at a deuteron energy of 700 keV under conditions corresponding to formation of the ground state of He^5 and found to be similar in shape to that obtained at 900 keV.

4. Interpretation of the Energy Distributions

The structure of the energy distributions is dependent on the kinetics of the break-up of Be^{9*}. For reaction (a) the energy E_1 of α_1 is related to the excitation of He^5, $Ex(He^5)$, by

$$E_1 = a_1 - a_2 Ex(He^5) \qquad \text{(i)}$$

and the energy E_2 of α_2 is related to E_1 and θ by

$$E_2 = b_1 + b_2 E_1 - b_3 \sqrt{b_4 E_1 - E_1^2} \cos\theta \qquad \text{(ii)}$$

where θ is the angle between the direction of recoil of He^5 and the direction of emission of α_2 in the system with He^5 at rest. The a's and b's are constants depending only on the total energy released from the break-up of Be^{9*} and on the energy of the incident deuteron.

The spin of the intermediate nucleus, Be^{9*}, is obtainable from knowledge of the angular correlation between the alpha-particles. From equation (ii) it follows that the angular correlation is given by the energy distribution of α_2 when E_1 is fixed. Both α_1 and α_2 are observed in counter II but, because of the peak due to α_1 at 8.25 MeV in Fig. 2, the energy distribution recorded in coincidence with 8.2 MeV alpha-particles (fig. 3) is mainly due to the detection of α_2. From this distribution the form $\sin^2\theta$ is obtained as a good approximation to the true angular correlation-function.

The spin of the ground state of He^5 is [2,3] $\frac{3}{2}$. Since the angular distribution of α_1 has been observed to be isotropic to within an estimated experimental error of 10 percent [6] it is assumed that the incoming deuterons have zero orbital angular momentum. Conservation of spin and parity allows α_1 to be emitted with either 0, 2 or 4 units of orbital angular momentum l. If only a single compound nucleus level of Be^{9*} is involved then, from the measured angular correlation, the spin of this level is determined [6] as $\frac{5}{2}$. For this spin the calculated correlation is of the form $1+A\sin^2\theta$ where A is a function of the partial widths for the emission of α_1 with $l = 2$ or $l = 4$. These widths have been calculated using the theory of Wigner and Eisenbud [7] and tables of Coulomb wave functions (Bloch *et al.* [8]). The ratio of the reduced widths has been chosen to give a maximum value for A. At $E_1 = 8.2$ MeV the maximum value for A is 7.0 and the ratio of the reduced widths for $l = 2$ and $l = 4$ is 7.8; using the same ratio at $E_1 = 6.78$ MeV the value for $A = 7.4$. Thus, as it is not possible on the basis of the

present observations to distinguish between angular correlations of the form $\sin^2\theta$ and $1+7\sin^2\theta$, the former has been adopted for both the energy distributions. The first excited state of He^5 has [2, 3]) spin $\frac{1}{2}$ and the transitions through this state give rise to an isotropic angular correlation.

It can be shown from (i) and (ii) that for each α_1 of energy E_1 detected in counter II there are unique values for Ex(He^5) and θ. If the effect of the angular correlation is allowed for, the energy distribution of α_1 may be directly related to the relative probability for the formation of He^5 at given excitations. These results may in turn be compared with the properties of He^5 obtained from the scattering of neutrons in He^4. The scattering cross section for neutrons in He^4 must first be related to the probablility for the formation of He^5 at given excitations through the break-up of Be^{9*}. We can write the contribution to the scattering cross section from each state in He^5 as

$$\sigma_{n\alpha} = \frac{1}{K^2} \frac{\Gamma_b \; \Gamma_b}{\text{resonance denominator}} \qquad \text{(iii)}$$

where K is the relative wave number of the n+He^4 system and Γ_b is the partial width for $He^5 \rightarrow$ n+He^4. The resonance denominator depends only on the energy of the n+He^4 system and the properties of He^5. It seems likely on the basis of calculations by Welton [9]) that the probability for the formation of He^5 at a given excitation through the irreversible reaction Li^7(d,α) He^5 can be written as

$$\sigma_{d\alpha} = \frac{1}{K_\alpha^2} \frac{\Gamma_a \; \Gamma_b}{\text{resonance denominator}} \qquad \text{(iv)}$$

where K_α is the relative wave number of the He^4+He^5 system and Γ_a is the partial width for $Be^{9*} \rightarrow He^4$+He^5. The resonance denominator and Γ_b are identical to those used in (iii). Both K and K_α are known and the energy dependence of Γ_a and Γ_b can be calculated from the tables of Coulomb wave functions (Bloch *et al.* [8])). In calculating Γ_a for the ground state transitions the ratio of the reduced widths has been taken as 7.8; for the excited state transitions only α_1 with $l = 2$ are involved and the reduced width for these transitions has been assumed equal to that for the emission of α_1 with $l = 2$ in the formation of the ground state.

The cross sections, $\sigma_{n\alpha}$, have been taken from the data of Adair [2]) and Seagrave [3]) for the ground and first excited states and the sum

is shown as curve A in fig. 5. The corresponding $\sigma_{d\alpha}$ has been calculated by combining (iii) and (iv) and is shown as curve B in fig. 5. It is possible that the factor K_α^{-2} in (iv) should in fact be replaced by K_α^{-1}, however this would only introduce an error of 4 per cent between the extremes of the range of excitation of He^5 considered. Only the relative probability for the formation of He^5 was calculated since the absolute values of the reduced widths for the emission of α_1 are not known.

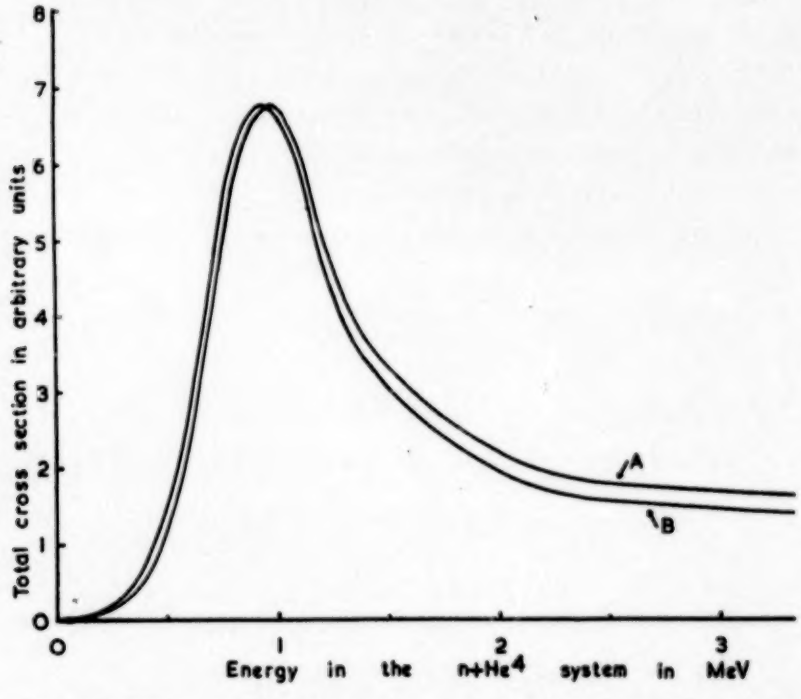

Fig. 5. Curve A: The contribution to the scattering cross section of neutrons in He^4 from the ground and first excited states of He^5. Curve B: The relative probability for the formation of He^5 at given excitation energies by the reaction $Li^7(d, \alpha)He^5$.

The energy distributions to be expected in the present observations have been calculated from curve B and the angular correlations referred to above. These distributions are presented as full curves in figures 3 and 4 and those parts of each due only to the detection of α_1 in counter II are shown as dashed curves. The relative magnitudes of the calculated distributions have been determined by the kinetics of the break-up of Be^{9*} and by the relative probability for the formation of He^5 as given by curve B at the two energies chosen for counter I. The total area under the full curve in fig. 3 has been made equal to the area under a smooth

curve drawn through the experimental points. The finite resolution of counters I and II and the distortion of the distributions due to the different angles at which the alpha-particles enter counter II have been taken into account. The centre of mass motion has been neglected in the break-up of Be^{9*}.

The group at the low energy end of each observed distribution is spurious and came about in the following way. The output from counter II was gated by pulses from the coincidence circuit and the gated output then recorded by the pulse height analyser. However due to some feed through of the pulses from the coincidence circuit a spurious signal was induced at the input to the analyser and all smaller pulses from counter II were recorded as if of the same height as the spurious signal. There is also a discrepancy of some 6 percent in energy between the observed and calculated distributions. The gas gain of counter II drifted by as much as 4 percent during a run. To correct for this drift the calibration was carried out at frequent intervals and each run was normalised to a standard energy scale. The error in making a number of such corrections for each distibution could account for the above discrepancy.

5. Conclusion

The results of the measurement of Adair [2]) and Seagrave [3]) on the scattering of neutrons in He^4 indicate a pronounced resonance due to the ground state of He^5 centred about a neutron energy of 1.25 MeV and a broad background due to the excited state with a maximum cross section at a neutron energy of approximately 8 MeV. Despite the uncertainty in energy the overall agreement between the observed and calculated distributions in the present experiment indicates that the formation of He^5 through reaction (a) is similar to its formation by the scattering of neutrons in He^4 but leads to no new information. The formation of He^5 has also been studied previously in the reactions T(t, n)He^5 (Leland and Agnew [10])), T(He^3 p)He^5 (Moak [11])), He^4(d, p)He^5 (Burge *et al.* [12])), Li^6(n, d)He^5 and Li^7(n, t)He^5 (Frye [13])) and Li^6(d, He^3)He^5 (Levine *et al.* [14])). In all cases except the first the results are consistent with the neutron scattering experiments and lead to no new information.

If Be^{9*} breaks up through reaction (b), the energies E_1 and E_2 of the alpha-particles and the excitation energy of Be^8, Ex(Be^8).

are related by

$$E_1+E_2 = C_1+C_2 \cdot \mathrm{Ex}(\mathrm{Be}^8). \qquad \text{(v)}$$

Alpha-particles from all Ex(Be^8) between 9.5 and 15.7 MeV would have been observed in the present energy distributions. Since the spurious low energy groups appeared at the same energy in each of the measured distributions they could not be due to the formation of a level in Be^8. If Be^{9*} breaks up through reaction (c) alpha-particles with a continuous energy distribution would be observed in each case. It is possible that the part of the distributions attributed to the first excited state of He^5 could be explained in terms of a three body break-up. Taking all experimental points the deviation from the calculated curves is approximately 20 percent If allowances were made for the discrepancy in the energy scales this figure would be halved. We conclude that the decay of Be^{9*} through reactions (b) and (c) contributed less than one tenth of the disintegrations observed in the present experiment.

This work was carried out as part of the research programme of this laboratory under Professor E. W. Titterton. The author wishes to record his gratitude for the generous help and encouragement given to him by Dr. P. B. Treacy throughout all phases of the experiment.

References

1) F. Ajzenberg and T. Lauritsen, Rev. Mod. Phys. **27** (1955) 77
2) R. K. Adair, Phys. Rev. **86** (1952) 155
3) J. D. Seagrave, Phys. Rev. **92** (1953) 1222
4) A. C. Riviere and P. B. Treacy, Aust. Jour. Phys. **8** (1955) 408
5) A. P. French and P. B. Treacy, Proc. Phys. Soc. A **64** (1951) 452
6) P. B. Treacy, Ph. D. Thesis, Cambridge University, (1951)
7) E. P. Wigner and L. Eisenbud, Phys. Rev. **72** (1947) 29
8) I. Bloch, M. H. Hull, A. A. Broyles, W. G. Bouricius, B. E. Freeman and G. Breit, Rev. Mod. Phys. **23** (1951) 147
9) T. A. Welton, Phys. Rev. **95** (1954) 302
10) W. T. Leland and H. M. Agnew, Phys. Rev. **82** (1951) 559
11) C. D. Moak, Phys. Rev. **92** (1953) 383
12) E. J. Burge, H. B. Burrows, W. M. Gibson and J. Rotblat, Proc. Roy. Soc. A **210** (1951) 534
13) G. M. Frye, Phys. Rev. **93** (1954) 1086
14) S. H. Levine, R. S. Bender and J. H. McGruer, Phys. Rev. **97** (1955) 1249

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POLARIZATION OF FAST PROTONS SCATTERED BY NUCLEI WITH INCLUSION OF THE ELECTRO-MAGNETIC SPIN-ORBIT COUPLING

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Abstract: Calculations have been made on the polarization of protons scattered by carbon, aluminium and iron at 130 MeV, with special attention to the spin-orbit coupling coused by the electromagnetic interaction between the nucleus and the incident nucleon. The calculations show that the effect of this interaction is noticeable for small scattering angles up to around 10° at this energy. The effect should be more pronounced in the case of neutron scattering.

The polarization of protons scattered by nuclei has been treated by several authors assuming a spin-orbit interaction in addition to the spin-independent central potential of the nucleus. The agreement with experiments has in the main been satisfactory for not too light nuclei, provided the Coulomb scattering is taken into account.

Another cause for the polarization has been pointed out by Schwinger [1]) and treated by him in the case of neutrons scattered by nuclei. Schwinger considers the coupling between the magnetic moment of the neutron and the nuclear Coulomb field in which the neutron is moving. The same effect with the reversed sign should appear for a proton. As the polarization caused by this coupling should be strongest for small scattering angles, the effect is to some extent destroyed — in the case of protons — by the strong forward Coulomb scattering. For a more detailed comparison of the experimental and theoretical polarization curves, however, this effect might give noticeable corrections to the polarization caused by the nuclear forces. †

The interaction between the nucleon magnetic moment and the

† During the preparation of this manuscript, there has appeared a paper by W. Heckrotte [2]), where the same effect is treated for protons scattered by carbon at 300 MeV.

nuclear Coulomb field gives a contribution to the Hamiltonian of the form

$$H' = \frac{ge\hbar}{4Mc}\,\boldsymbol{\sigma}\cdot\left(\frac{1}{c}\mathbf{v}\times\mathbf{E}\right) = -\frac{ge\hbar}{4M^2c^2}\,\boldsymbol{\sigma}\cdot(\mathbf{E}\times\mathbf{p}), \qquad (1)$$

where $\mathbf{p} = M\mathbf{v}$ is the momentum of the incident nucleon, $\mathbf{E}$ is the nuclear Coulomb field and g is the Landé factor (5.59 for protons). To obtain the above expression (the Larmor precession), one has to make a transformation from a coordinate system fixed in the nucleus to a system fixed in the incident nucleon. As the proton coordinate system is accelerated in the Coulomb field there appears, when transforming back to the nuclear system, another spin-orbit term, the Thomas precession [3]). This is easily taken into account by putting (for protons) $g-1$ instead of g in equation (1) Writing $\mathbf{r}\times\mathbf{p} = \hbar\mathbf{l}$ one gets

$$H' = F(r)\mathbf{l}\cdot\mathbf{s} \qquad (\mathbf{s} = \tfrac{1}{2}\boldsymbol{\sigma}). \qquad (2)$$

The function $F(r)$ is given by the assumption that the Coulomb field is the same as that from a uniform charge distribution within a radius R_c smaller than or equal to the nuclear radius.

In the following calculations the nucleus was represented by a spherical, complex potential with linearly sloping sides and with a spin-orbit coupling of the type $(1/r)(\mathrm{d}V/\mathrm{d}r)$, where V stands for the real part of the nuclear potential [4, 5]). In one case the spin-independent part of the potential was represented by a cut-off harmonic oscillator well [6]). Thus:

$$\begin{aligned} V_p &= V_p' - C_1\mathbf{l}\cdot\mathbf{s}, \quad \text{for } r < R_c \\ V_p &= V_p' - C_2\frac{1}{r^3}\mathbf{l}\cdot\mathbf{s} \quad \text{for } R_c < r < R \\ V_p &= \frac{Ze^2}{r} - C_2\frac{1}{r^3}\mathbf{l}\cdot\mathbf{s} \quad \text{for } R < r; \end{aligned} \qquad (3)$$

V_p' is the nuclear and Coulomb potential for the incident protons. The real and imaginary parts of the nuclear potential were chosen to give total and inelastic scattering cross sections in accordance with experiments (cf. ref. [4, 5]). The second term is the potential due to the Coulomb spin-orbit force. It is, of course, very small compared to the non-electromagnetic spin-orbit coupling in the nucleus. R is the maximum nuclear radius and

$$C_1 = \frac{(g-1)Ze^2\hbar^2}{2M^2c^2R_c^3}, \qquad C_2 = \frac{(g-1)Ze^2\hbar^2}{2M^2c^2},$$

The scattered wave can be written

$$\psi_{sc} = (f(\theta) + ig(\theta)\boldsymbol{\sigma}\cdot\mathbf{n})\,\chi_{inc}\frac{e^{ikr}}{r}. \tag{4}$$

The phases in the formulae for the amplitudes (eq. (4) in ref. [6])) were calculated by means of the WKB method in the limit of high energies [7]).

The sums over l in the formulae for the scattering amplitudes are replaced by integrals. The Legendre functions are then written in terms of Bessel functions [8]):

$$P_l^{(s)}(\cos\theta) \approx \frac{l!}{(l-1)!}\left(\frac{\theta}{\sin\theta}\right)^{\frac{1}{2}} J_s((l+\tfrac{1}{2})\theta). \tag{5}$$

The expressions for the scattering amplitudes f and g can now be written

$$f(\theta) = f_c(\theta) - \frac{i}{2}RL\left(\frac{\theta}{\sin\theta}\right)^{\frac{1}{2}}\int_0^\infty \{e^{2i\delta_l^+} + e^{2i\delta_l^-} - 2e^{2i\eta_l}\}J_0(L\theta x)\,dx$$

$$g(\theta) = -\frac{i}{2}RL\left(\frac{\theta}{\sin\theta}\right)^{\frac{1}{2}}\int_0^\infty \{e^{2i\delta_l^-} - e^{2i\delta_l^+}\}J_1(L\theta x)\,dx, \tag{6}$$

where $f_c(\theta)$ is the amplitude for scattering by a point charge. $\delta_l^\pm$ are the phase-shifts caused by the potential (3) with spin direction parallel and antiparallel to the orbital angular momentum, and η_l is given by $(Ze^2k/2E)\ln(l+\frac{1}{2})$ in the present approximation. Further we have written $L = kR$ and $x = L^{-1}(l+\frac{1}{2})$.

The range of integration in (6) can be split into two parts: 0 to l and l to infinity. In the latter interval the phase-shifts $\delta_l^\pm$ are due to the usual Coulomb potential and Coulomb spin-orbit coupling only. By making the substitution $y = L\theta x$ and inserting the analytical expressions for the phase-shifts, the extra contributions to the scattering amplitudes f and g due to the Coulomb spin-orbit coupling can be written

$$f'(\theta) = i\frac{R}{L}\left(\frac{\theta}{\sin\theta}\right)^{\frac{1}{2}}\frac{2(k^3C_2)^2}{(4E)^2}e^{-2in\ln\theta}\int_{L\theta}^\infty y^{2in-1}J_0(y)\,dy;$$

$$g'(\theta) = -\frac{R}{L\theta}\left(\frac{\theta}{\sin\theta}\right)^{\frac{1}{2}}\frac{2k^3C_2}{4E}e^{-2in\ln\theta}\int_{L\theta}^\infty y^{2in}J_1(y)\,dy; \tag{7}$$

where $n = Ze^2k/2E$, and the approximation

$$\cos 2\delta_i^{\pm} - 1 \approx -\tfrac{1}{2}(2\delta_i^{\pm})^2, \qquad \sin 2\delta_i^{\pm} \approx 2\delta_i^{\pm}$$

have been made. These approximations do not play any significant rôle in these calculations. As can be expected the contribution f' to f is small and can be omitted. (The smallness of f' is in accordance with a Born approximation treatment, where f' is zero †). The integrals can be evaluated exactly [9]).

The calculations have been made for protons scattered by aluminium and carbon at 135 MeV and by iron at 150 MeV. The

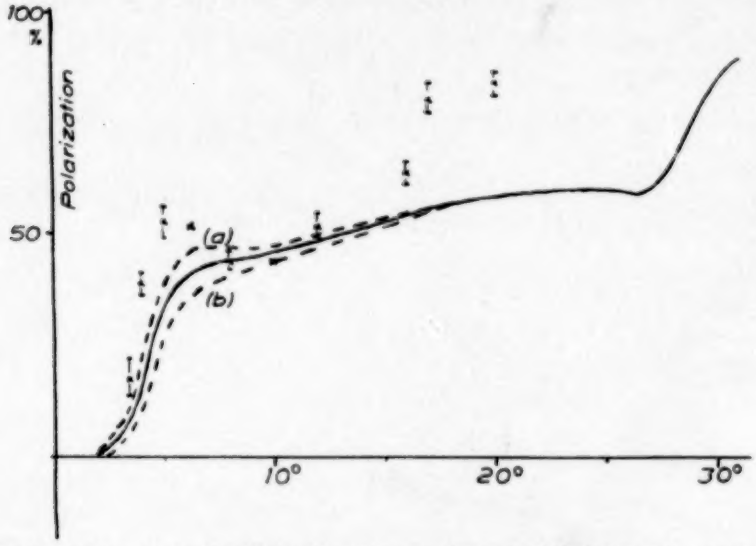

Fig. 1. Polarization of protons scattered by carbon at 130 MeV. The dotted curves show the results when the Coulomb spin-orbit coupling is included, (a) if the nuclear non-electro-magnetic spin-orbit interaction is given the same sign as in the shell model, (b) for reversed sign of the nuclear spin-orbit coupling (in this case the sign of the polarization should also be reversed).

results are shown in figures 1—3. The crosses are the experimental results at 135 MeV from Harwell [10]).

The full curve in fig. 1 shows the theoretical results for carbon, when only the nuclear spin-orbit interaction is taken into account. The nuclear potential is chosen to be a complex potential with linearly sloping sides. The outer radius R is chosen to be 4.35×10^{-13}

† If the scattering amplitudes f and g for the nuclear scattering are known one could think of using the Born approximation to correct for the Coulomb spin-orbit interaction. One does then get contributions to the amplitude g only. The results are of the right order of magnitude for carbon, but the approximation breaks down for iron, and this can be seen to be connected with a strong increase in the phase-shifts, when going form carbon to iron.

cm. The inner radius, which limits the constant part of the potential, is 0.3 R. The potential depth in MeV at the centre, which reproduces the experimental cross sections is $19+15i$. The dotted curve (a) shows the results when the Coulomb spin-orbit interaction is taken into account. If the nuclear spin-orbit coupling is given the opposite sign (opposite to that required by the shell model), curve (b) is obtained (the sign of the polarization is also reversed). There are two obvious discrepancies between the experimental and theoretical polarization curves. The first appears at 5° scattering angle, where the experimental values have a maximum. A better agreement at 5° can be obtained by changing the parameters of the nuclear spin-independent potential, and taking into account the Coulomb spin-orbit interaction as in curve (a). The second discrepancy, however, at angles greater than 15° does not seem to be eliminated by any reasonable choice of the parameters. One reason for this discrepancy could be effects associated with the strong inelastic scattering that appears at these angles [11]) †. Until these disagreements, especially

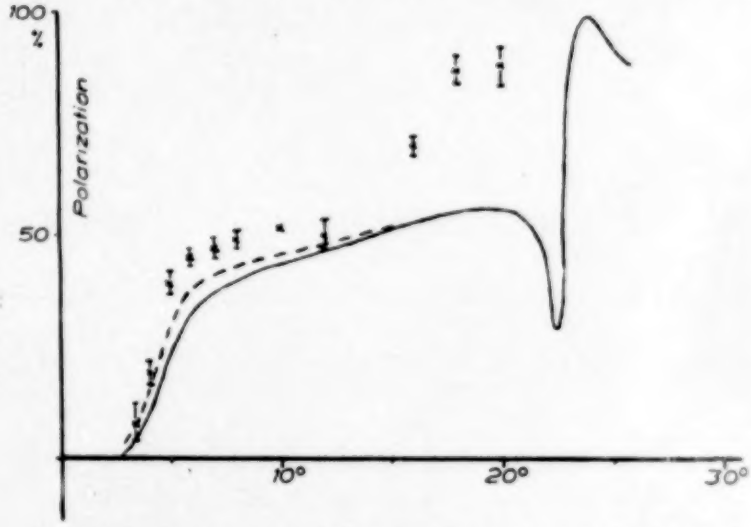

Fig. 2. Polarization of protons scattered by aluminium at 130 MeV. The dotted curve shows the effect of the Coulomb spin-orbit coupling.

the one around 15°, have been explained there would be no meaning in making a too detailed comparison between the experimental and theoretical curves in fig. 1. In any case, however, up to about 10° scattering angle the contribution from the Coulomb spin-orbit coupling is not negligible. The uncertainties referred to above do not

† Similar measurements have been made at an energy around 180 MeV at the Gustaf Werner Institute for Nuclear Chemistry, Uppsala.

warrant any definite conclusions about the sign of the nuclear spin-orbit coupling to be made on the basis of the difference between curves (a) and (b), but curve (a) seems more reasonable than curve (b).

For aluminium (fig. 2) the spin-independent part of the nuclear potential was chosen to be a cut-off harmonic oscillator. The calculation is based on Köhler's calculation (Cf. ref. [6])). The same disagreements between the theoretical and experimental results for carbon seem to be present for aluminium. The effect of the Coulomb spin-orbit interaction (dotted curve) is recognizable up to about 10°.

The calculations for iron were made for a potential with linearly sloping sides as in ref. [5]). The difference in energy between experiments and theory, 135 and 150 MeV respectively, cannot be the only reason for the difference in polarization at small angles.

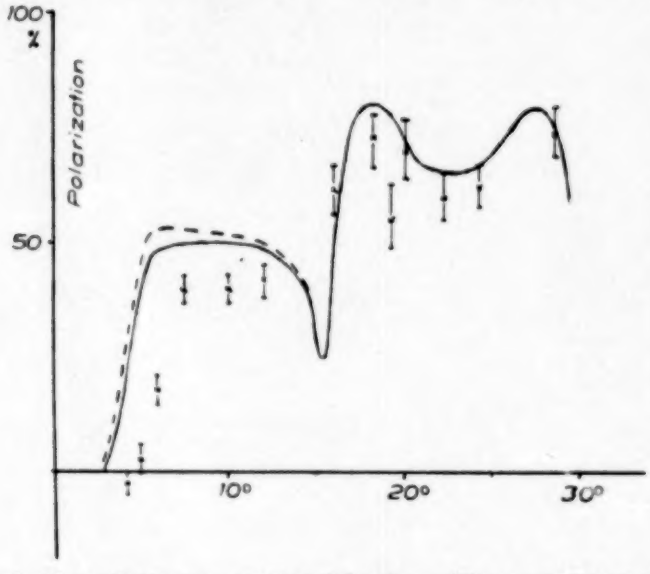

Fig. 3. Polarization of protons scattered by iron. The experimental values at an energy of 130 MeV, the calculated curve at 150 MeV.

The dotted curve shows that also in this case does the Coulomb spin-orbit interaction influence the results up to around 10°.

The calculations show that for proton polarization the Coulomb spin-orbit interaction gives noticeable contributions for small scattering angles up to about 10° at an energy around 130 MeV. The pure Coulomb scattering, however, makes the polarization effects small for small angles. One should therefore obtain more easily observable effects in neutron scattering. In the case of iron, for instance, the neutron polarization curve at 150 MeV calculated

with this nuclear model, will vary almost linearly from 0 % at 0° to about 60 % around 15°. If the Coulomb spin-orbit interaction is included the polarization curve will deviate appreciably from the linear behaviour for small scattering angles.

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References

1) J. Schwinger, Phys. Rev. **73** (1948) 407
2) W. Heckrotte, Phys. Rev. **101**, (1956) 1406
3) L. Rosenfeld, Nuclear Forces (North Holland Publishing Co. 1948) p. 316
4) W. J. Swiatecki, Phys. Rev. **98** (1955) 204
5) T. Eriksson, Nuovo Cimento **2** (1955) 907
6) S. Köhler, Nuovo Cimento **2** (1955) 911
7) S. Fernbach, R. Serber and T. B. Taylor, Phys. Rev. **75** (1949) 1352
8) H. and B. S. Jeffreys, Mathematical Physics (University Press, Cambridge 1946) p. 617
9) G. N. Watson, Theory of Bessel Functions, 2nd Ed. (University Press, Cambridge 1944) p. 350
10) B. Rose, Proc. of the Fifth Rochester Conf. on High Energy Nuclear Physics, and private communication
11) K. Strauch and W. F. Titus, Phys. Rev. **95** (1955) 854

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ISOBARIC SPIN AND DESCRIPTION OF ANTIPARTICLES

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Abstract: Some remarks on the d'Espagnat-Prentki interpretation of the Gell-Mann model are made in connection with a symmetrical description of particles and antiparticles by means of quantized fields and antifields.

1. Introduction

There is no doubt that in a satisfactory theory the particles and their antiparticles must be described on the same footing. This can be done for fermions as well as bosons with *isobaric* spin by introducing the antifields by which the antiparticles are described in the same fashion as the particles by the original fields. We shall see that the definition of these antifields can be given by formally the same operation in all cases of the fields considered in the d'Espagnat-Prentki scheme [1]), if we use unspecified representation of the ordinary and isobaric spin matrices and keep all equations and expressions invariant under the change of this representation by unitary transformations.

The antifields, generally, are not identical with the charge-conjugate fields. The notion of the antiparticle is independent of any special type of interaction†. The antifields can therefore be defined *invariantly* with respect to coordinate transformation *in the isobaric space* as well as in space-time. On the other hand, the notion of charge-conjugation is closely connected with the special nature of the interaction with the electromagnetic field which introduces an anisotropy or preferred direction in the isobaric space. Henceforth the definition of the charge-conjugates of the fields (and antifields) need not (and cannot) be isoinvariant. Nevertheless,

† By definition, the annihilation (creation) operators of particles (anti-particles) are always associated with the positive (negative) frequency terms in the Fourier development of the field under consideration.

the charge-conjugates can also be defined quite generally (i.e. by the same operation) for all types of fields included in the d'Espagnat-Prentki set and the invariance of the whole theory under charge conjugation can be shown.

Interesting consequences can be drawn from the fact that *fields* and *antifields* have an equal right to be considered as primary ones. We shall see that we must settle the behaviour of the different baryon fields not only for the reflections in the isobaric space (i e. their relative *isobaric* parities) but also for reflections in the *ordinary* space. Difficulties as to the selection of the two sub-classes called "baryons" and "antibaryons" can be avoided only if the same (relative) *ordinary* parity is assigned to all the baryon fields. Moreover, their antifields must have *opposite* ordinary parity. The selection of the two sub-classes of baryons and antibaryons can then be done on the same principle as the selection of the two sub-classes of isofermions and anti-isofermions, i.e. according to the *ordinary* and *isobaric* parities of the corresponding fields respectively.

2. Fields and Antifields

We assume for the sake of simplicity that all the baryons have spin $\frac{1}{2}$ and the θ-mesons have spin 0 or 1. Then it is reasonable to assume further that the fields included in the d'Espagnat-Prentki set satisfy (in the presence of an external electromagnetic field and the absence of other interactions) the wave equations

$$[\gamma_\mu(\partial_\mu - ieA_\mu q) + m(\mathrm{N})]\psi_{(\mathrm{N})} = 0, \tag{1}$$

$$[\gamma_\mu \partial_\mu + m(\Lambda)]\psi_{(\Lambda)} = 0, \tag{2}$$

$$[\gamma_\mu(\partial_\mu - ieA_\mu \zeta_3) + m(\Sigma)]\psi_{(\Sigma)} = 0, \tag{3}$$

$$[\gamma_\mu(\partial_\mu - ieA_\mu \dot{q}) + m(\Xi)]\psi_{(\Xi)} = 0, \tag{4}$$

$$[\beta_\mu(\partial_\mu - ieA_\mu \zeta_3) + m(\pi)]\psi_{(\pi)} = 0, \tag{5}$$

$$[\beta_\mu(\partial_\mu - ieA_\mu q) + m(\theta)]\psi_{(\theta)} = 0. \tag{6}$$

Here $q = \frac{1}{2}(\tau_3 + 1)$ and $\dot{q} = \frac{1}{2}(\tau_3 - 1)$. The $\gamma_\mu = \gamma_\mu^+$ ($\mu = 1, 2, 3, 4$) are the Dirac matrices and $\beta_\mu = \beta_\mu^+$ are the Duffin-Kemmer matrices. In the pion equation (5) the β's are of course the 5×5 matrices belonging to spin 0. The $\tau_i = \tau_i^+$ ($i = 1, 2, 3$) are the Pauli matrices (in arbitrary representation) and $\zeta_i = \zeta_i^+$ are 3×3 matrices representing the isobaric spin 1. They are defined only

by their irreducibility and their commutation relations (similar to D.-K. relations):

$$\zeta_i\zeta_j\zeta_k+\zeta_k\zeta_j\zeta_i=\delta_{ij}\zeta_k+\delta_{kj}\zeta_i,$$
$$[\zeta_1,\ \zeta_2]=i\zeta_3 \text{ (cycl. 1, 2, 3).}$$

The components of the ψ's will be denoted generally as $\psi_{\alpha a}(x)$. The index α runs from 1 to 4 for baryons, from 1 to 5 for pions and from 1 to 5 or 1 to 10 for θ-mesons. The index a takes the values 1, 2 for N, Ξ and θ, the values 1, 2, 3 for Σ and π and is absent in the case of Λ. The adjoint wave functions are defined by $\bar{\psi}=\psi^+\gamma_4$ (in the cases N, Λ, Σ, Ξ) or by $\bar{\psi}=\psi^+(2\beta_4^2-1)$ (in the cases π and θ). The function $\bar{\psi}_{(\mathrm{N})}$ for example fulfils the equation

$$[\gamma_\mu^{\mathrm{T}}(\partial_\mu+ieA_\mu q^{\mathrm{T}})-m(\mathrm{N})]\,\bar{\psi}_{(\mathrm{N})}^{\mathrm{T}}=0 \qquad (\overline{1^{\mathrm{T}}})$$

and similar equations are valid in other cases. The commutation relations in the case of baryons are as usual

$$\{\psi_{\alpha a}(\mathbf{x},t),\ \psi^+_{\beta b}(\mathbf{x}',t)\}=\delta_{\alpha\beta}\delta_{ab}\delta(\mathbf{x}-\mathbf{x}'),\ \{\psi,\ \psi\}=\{\psi^+,\ \psi^+\}=0. \quad (7)$$

In the case of θ-mesons we have [2])

$$[(\beta_4^2\psi(\mathbf{x},t))_{\alpha a},\ (\psi^+(\mathbf{x}',t)\beta_4^2)_{\beta b}]=(\beta_4)_{\alpha\beta}\,\delta_{ab}\,\delta(\mathbf{x}-\mathbf{x}'),$$
$$[\psi,\ \psi]=[\psi^+,\ \psi^+]=0. \quad (8)$$

For the field $\psi_{(\pi)}$ only the first row of (8) remains valid because there is, as we shall see, an auxiliary relation between $\psi_{(\pi)}$ and $\psi^+_{(\pi)}$ so that the commutation relations between the components of $\psi_{(\pi)}$ are determined by the first row of (8) already.

The fields $\psi_{(\mathrm{N})}$, $\psi_{(\Lambda)}$, $\psi_{(\Sigma)}$, $\psi_{(\Xi)}$ describe the *baryons* N^+, N^0; Λ^0; Σ^+, Σ^0, Σ^-; Ξ^0, Ξ^- as *particles* and the corresponding *antibaryons* as *antiparticles*. The field $\psi_{(\theta)}$ describes θ^+, θ^0 as *particles* and $\bar{\theta}^+\equiv\theta^-$ and $\bar{\theta}^0$ as *antiparticles*. In the case $\psi_{(\pi)}$ the particles $\pi^\pm$, π^0 are *identical* with the equally charged antiparticles $\overline{\pi^\mp}$, $\overline{\pi^0}$ (in virtue of the auxiliary condition (18a) to be imposed on $\psi_{(\pi)}$) and must not be counted separately (see the expressions for $L_{(\pi)}$ and $J_\mu^{(\pi)}$ as given by (19) and (20)).

However, the same physical situation can also be described by introducing instead of the fields ψ their antifields $\dot{\psi}$ which are defined as follows:

First, we introduce four unitary matrices C, B, T, Z satisfying the relations

$$C\gamma_\mu^{\mathrm{T}}=-\gamma_\mu C,\ B\beta_\mu^{\mathrm{T}}=-\beta_\mu B,$$
$$T\tau_i^{\mathrm{T}}=-\tau_i T,\ Z\zeta_i^{\mathrm{T}}=-\zeta_i Z. \qquad (9)$$

The matrices C and T are anti-symmetrical whereas B and Z are symmetrical [3]).

Now we put †

$$\begin{aligned} \dot{\psi}_{(N)} &= CT\overline{\psi}^{T}_{(N)}, \quad \dot{\psi}_{(\Lambda)} = C\overline{\psi}^{T}_{(\Lambda)}, \quad \dot{\psi}_{(\Sigma)} = CZ\overline{\psi}^{T}_{(\Sigma)}, \\ \dot{\psi}_{(\Xi)} &= CT\overline{\psi}^{T}_{(\Xi)}, \quad \dot{\psi}_{(\pi)} = BZ\overline{\psi}^{T}_{(\pi)}, \quad \dot{\psi}_{(\theta)} = BT\overline{\psi}^{T}_{(\theta)}. \end{aligned} \tag{10}$$

It is easy to verify that the wave equations satisfied by the antifields $\dot{\psi}$ result from the respective equations (1)—(6) simply by the exchange $q \rightleftarrows \dot{q}$. (Notice especially this important circumstance that the difference between the wave equations for the antifields $\dot{\psi}$ and for the original fields ψ does not consist in the *sign of* e but rather in the *sign of the d'Espagnat-Prentki quantity* U.) Moreover the commutation relations (7) and (8) remain valid also for the corresponding antifields. Thus it is clear that so far we can *consider with equal right the antifields* $\dot{\psi}$ *as the primary ones* instead of the original fields ψ. (It will be shown below that this remains valid also if we consider the possible "strong interactions" between the baryons and mesons.) For example the expressions for the *Lagrangian* and *current four-vector* of *nucleons and anti-nucleons* can be written in the symmetric froms

$$L_{(N)} = -\tfrac{1}{2}\{\overline{\psi}[1]\psi + \dot{\psi}[\dot{1}]\dot{\psi}\} \tag{11}$$

$$J^{(N)}_{\mu} = \frac{\partial L_{(N)}}{\partial A_{\mu}} = \frac{ie}{2}\{\bar{\psi}\gamma_{\mu}q\psi + \overline{\dot{\psi}}\gamma_{\mu}\dot{q}\dot{\psi}\}. \tag{12}$$

From (12) the total electric charge is

$$eQ = -i\int J_4 \mathrm{d}V = e\left(I_3 + \frac{U}{2}\right) \tag{13}$$

where

$$I_3 = \int \frac{1}{2}\left(\bar{\psi}\gamma_4\frac{\tau_3}{2}\psi + \overline{\dot{\psi}}\gamma_4\frac{\tau_3}{2}\dot{\psi}\right)\mathrm{d}V \tag{14}$$

† If we change the representation of the γ's, β's, τ's and ζ's by *unitary* transformations

$$\gamma_\mu \to S\gamma_\mu S^{+}, \quad \beta_\mu \to U\beta_\mu U^{+}, \quad \tau_i \to s\tau_i s^{+}, \quad \zeta_i \to u\zeta_i u^{+}$$

$$\psi_{(N)} \to Ss\psi_{(N)}, \quad \psi_{(\Lambda)} \to S\psi_{(\Lambda)}, \quad \psi_{(\Sigma)} \to Su\psi_{(\Sigma)}$$

$$\psi_{(\Xi)} \to Ss\psi_{(\Xi)}, \quad \psi_{(\pi)} \to Uu\psi_{(\pi)}, \quad \psi_{(\theta)} \to Us\psi_{(\theta)},$$

then C, B, T and Z transform according to $C \to SCS^{T}$, $B \to UBU^{T}$, $T \to sTs^{T}$, $Z \to uZu^{T}$. The latter transformations conserve the *unitarity* and *symmetry* (or *anti-symmetry*) of B and Z (or C and T) just as well as the former ones conserve the *hermiticity* of the γ's, β's, τ's and ζ's (and *unitarity* of the γ's and τ's). The antifields (10) transform in the same manner as the corresponding original fields. All the above quoted transformations form groups.

and

$$U = \int \tfrac{1}{2}(\bar{\psi}\gamma_4\psi - \bar{\dot{\psi}}\gamma_4\dot{\psi})\,\mathrm{d}V. \tag{15}$$

Inserting from (10) for $\dot{\psi}_{(N)}$, $\bar{\dot{\psi}}_{(N)}$ (or conversely for $\bar{\psi}$ and ψ) and using (9) it is easily shown that all these expressions (11)—(15) are in agreement with the usual Heisenberg prescription in the theory of holes. The antifield $\dot{\psi}_{(N)}$, considered as a primary field, describes the *anti-nucleons* $\overline{N}^0$ and $\overline{N}^+ \equiv N^-$ as *particles* and the *nucleons* N^0, N^+ as *antiparticles*. Similar expressions and assignments are valid in other cases too.

3. Transformation Properties of Fields and Antifields

Now let us consider the transformation properties of the fields and antifields under rotations and reflections of axes in the isobaric space and under Lorentz transformations and reflections in the ordinary space.

First, as concerns the fields and antifields which are *isospinors* (cases N, Ξ, θ), we can see from (10) that the antifields have *opposite isobaric parity* to the corresponding fields. Indeed, the rules for the reflection on the plane perpendicular to the i-th axis are

$$\psi \to \pm\tau_i\psi, \quad \dot{\psi} \to \mp\tau_i\dot{\psi} \tag{16}$$

and for the total isobaric space reflection

$$\psi \to \pm i\psi, \quad \dot{\psi} \to \mp i\dot{\psi}. \tag{17}$$

The fields and antifields also have opposite values of the quantity U. Thus it is natural to *assign* to $\psi_{(N)}$, $\dot{\psi}_{(\Xi)}$ and $\psi_{(\theta)}$ (which have the same $U = +1$ according to (1), (4) and (6)) *the same relative isobaric parity* (say $+$) and to select the two sub-classes of *isofermions* (N, $\overline{\Xi}$, θ) and *anti-isofermions* ($\overline{N}$, Ξ, $\bar{\theta}$) according to the *isobaric* parity of the corresponding fields.

As to the antifields $\dot{\psi}_{(\Lambda)}$, $\dot{\psi}_{(\Sigma)}$ and $\dot{\psi}_{(\pi)}$, it is easily shown that their transformation properties in the *isobaric* space are exactly the same as those of the corresponding original fields (see Appendix). The particles belonging to these fields and antifields are "isobosons" according to the terminology of d'Espagnat and Prentki.

The transformation properties of $\psi_{(\pi)}$ and $\dot{\psi}_{(\pi)}$ are, in addition, the same also in space-time. Both $\psi_{(\pi)}$ and $\dot{\psi}_{(\pi)}$ also fulfil the same equations. Now, $\dot{\psi}_{(\pi)}$ describes the "antipions" $\overline{\pi^-}$, $\overline{\pi^0}$, $\overline{\pi^+}$ as *particles* and π^-, π^0, π^+ as their antiparticles. The complete identity of the

formal properties of $\psi_{(\pi)}$ and $\dot{\psi}_{(\pi)}$ means that the two members of the pairs $(\pi^+, \overline{\pi^-})$ etc. are physically indistinguishable and that our theory gives us a "two-fold description" of the same things. This can be avoided if we impose on $\psi_{(\pi)}$ the auxiliary condition

$$\dot{\psi}_{(\pi)} = \psi_{(\pi)} \tag{18}$$

i.e.

$$BZ\bar{\psi}^{\mathrm{T}}_{(\pi)} = \psi_{(\pi)} \tag{18a}$$

and then "abbreviate" the theory by putting

$$L_{(\pi)} = -\tfrac{1}{2}\bar{\psi}_{(\pi)}[5]\psi_{(\pi)} = -\tfrac{1}{2}\overline{\dot{\psi}}_{(\pi)}[5]\dot{\psi}_{(\pi)} \tag{19}$$

so that

$$J^{(\pi)}_{\mu} = \frac{\partial L_{(\pi)}}{\partial A_{\mu}} = \frac{ie}{2}\bar{\psi}_{(\pi)}\beta_{\mu}\zeta_3\psi_{(\pi)} = \frac{ie}{2}\overline{\dot{\psi}}_{(\pi)}\beta_{\mu}\zeta_3\dot{\psi}_{(\pi)}\,. \tag{20}$$

The condition (18) or (18a) and the factor $\frac{1}{2}$ in (19) and (20) ensure that π^+, $\overline{\pi^-}$ etc. are considered as identical and are *not counted separately*. The condition (18a) is equivalent to the usual condition of reality of the pseudoscalar-isopseudovector wave function $\varphi_i = \varphi_i^+$ as used in the more familiar form of the pion-theory (see Appendix).

As concerns the transformation properties of the remaining fields and antifields *in space-time*, the following can be said. They are the same for $\psi_{(\theta)}$ and $\dot{\psi}_{(\theta)}$. However, these two wave functions have opposite *isobaric* parities (and opposite U-values) and (therefore) fulfil different wave equations. Thus, especially, θ^0 and $\bar{\theta}^0$ *are different* (θ^0 belongs to $\tau_3 = -1$, $U = +1$, whereas $\bar{\theta}^0$ belongs to $\tau_3 = +1$, $U = -1$).

It remains to consider the *baryon fields* and *antifields* which, according to our assumption, are all Dirac spinors in space-time. For Dirac spinors the two well-known rules

$$\psi \to \pm\gamma_4\psi \quad (\dot{\psi} \to \mp\gamma_4\dot{\psi}) \tag{21a}$$

or

$$\psi \to \pm i\gamma_4\psi \quad (\dot{\psi} \to \pm i\gamma_4\dot{\psi}) \tag{21b}$$

for total space reflection are acceptable. However, if we try to choose the rule (21b), in which case the fields would have the same (ordinary) parity as their antifields, the following difficulties appear: Firstly the fields $\dot{\psi}_{(\Xi)}$ and $\psi_{(N)}$ (or $\psi_{(\Xi)}$ and $\dot{\psi}_{(N)}$) would have much more similar properties (e.g. the *same isobaric* parity, the *same U-value* and therefore the *same form of wave equation*) than the fields $\psi_{(\Xi)}$ and $\psi_{(N)}$ (or $\dot{\psi}_{(\Xi)}$ and $\dot{\psi}_{(N)}$). Thus it would be hard to conceive

why Ξ^0, Ξ^- belong to baryons (like N^+ and N^0) and $\overline{\Xi^-} \equiv \Xi^+$, $\overline{\Xi^0}$ to antibaryons (like $\overline{N}^0$, $\overline{N}^+ \equiv N^-$) rather than conversely. Secondly the formal properties of $\dot{\psi}_{(\Lambda)}$ and $\psi_{(\Lambda)}$ (and of $\dot{\psi}_{(\Sigma)}$ and $\psi_{(\Sigma)}$) would be completely the same (as in the case of $\dot{\psi}_{(\pi)}$ and $\psi_{(\pi)}$). Thus it would be hard to conceive what "causes" the (physically necessary) difference between Λ^0 and $\overline{\Lambda}^0$, or between Σ^+ and $\overline{\Sigma^-}$ etc.

These difficulties can be avoided if we choose the *rule* (21*a*) *for baryon fields* and assign the same *ordinary* parity to $\psi_{(N)}$, $\psi_{(\Lambda)}$, $\psi_{(\Sigma)}$ and $\psi_{(\Xi)}$. Then, in fact, the correct selection of the two sub-classes of baryons (N, Λ, Σ, Ξ) and antibaryons ($\overline{N}$, $\overline{\Lambda}$, $\overline{\Sigma}$, $\overline{\Xi}$) can be done according to the *ordinary* parity of the corresponding fields, i.e. on quite similar principles as the selection of the sub-classes "isofermions" and "anti-isofermions".

4. Strong Interactions between Baryons and Mesons

The Lagrangian of the type proposed by d'Espagnat and Prentki can be written in the form

$$\begin{aligned} L_{(\text{strong})} = g\{ & i(\bar{\psi}_{(N)}\gamma_5\tau_j\psi_{(N)})\varphi_{(\pi)j} + i(\bar{\psi}_{(\Sigma)}\gamma_5\zeta_j\psi_{(\Sigma)})\varphi_{(\pi)j} + \\ & + i(\bar{\psi}_{(\Xi)}\gamma_5\tau_j\psi_{(\Xi)})\varphi_{(\pi)j} + i(\bar{\psi}_{(N)}\tau_j\vartheta_{(\theta)})\chi_{(\Sigma)j} + \\ & + (\bar{\psi}_{(N)}\vartheta_{(\theta)})\psi_{(\Lambda)} + i(\bar{\psi}_{(\Lambda)}\gamma_5\chi_{(\Sigma)j})\varphi_{(\pi)j} + \\ & + \bar{\psi}_{(\Lambda)}(\vartheta^{\mathrm{T}}_{(\theta)}T^+\psi_{(\Xi)}) + i(\bar{\psi}_{(\Xi)}\tau_j T\vartheta^{+\mathrm{T}}_{(\theta)})\chi_{(\Sigma)j}\} \\ & + \text{Hermitian-conjugate expression.} \end{aligned} \tag{22}$$

We have assumed the same real coupling constant g for all terms. Further we are using (for the sake of simplicity) spin 0 (even parity) and scalar coupling for the θ-meson as well as pseudoscalar coupling for π-mesons. The definition of the quantities φ_j, χ_j and ϑ is given in the Appendix.

The Lagrangian $L_{(\text{st})}$ is a true scalar and isoscalar. It conserves besides the total isobaric spin $\mathbf{I}^2$, I_3 also the total quantity N (number of baryons minus number of antibaryons) and the total quantity U (number of isofermions minus number of anti-isofermions). Thus it also conserves the Gell-Mann "strangeness" $S = U - N$ and the total electric charge $Q = I_3 + \frac{1}{2}U$.

The total Lagrangian is the sum of terms like $L_{(N)}$, $L_{(\pi)}$ (see espressions (11), (19) etc., describing individual kinds of fields and antifields under the influence of the electromagnetic field) and of the expression $L_{(\text{strong})}$.

5. Invariance Properties of Lagrangian

The Lagrangian $L_{(st)}$ can also be written in the form

$$L_{(st)} = g\{\ldots\} - g\{\text{anti}\} \tag{22a}$$

where $\{\text{anti}\}$ denotes the expression in the curled bracket in (22) with antifields instead of original fields.

Hence $L_{(strong)}$ is invariant under the substitution

$$\psi \rightleftarrows \dot{\psi}, \quad g \to -g \tag{23}$$

As to the terms $L_{(N)}$, $L_{(\pi)}$ etc., they remain unchanged under (23) if we simultaneously make the exchange

$$q \equiv \tfrac{1}{2}(\tau_3+1) \rightleftarrows \tfrac{1}{2}(\tau_3-1) \equiv \dot{} \,. \tag{23a}$$

Thus the exchange of fields and antifields is related to the change of the sign of the "mesic charge" and of the quantity U. Of course, it also changes the sign of the quantity N.

Now, the total Lagrangian is invariant also under another important transformation, viz. the *charge conjugation*. To see this let us define the charge conjugate fields ψ' and antifields $\dot{\psi}'$ as follows: We perform the operation R_2 corresponding to the reflection on the plane perpendicular to the 2-nd axis in the isobaric space and then put

$$\begin{aligned} \psi' &= -R_2\dot{\psi}, \quad \dot{\psi}' = -R_2\psi, \\ A'_\mu &= -R_2\mathring{A}_\mu = -R_2 A_\mu = -A_\mu. \end{aligned} \tag{24}$$

From the equations (1)—(6) we can obtain the equations ($\dot{1}$)—($\dot{6}$) satisfied by $\dot{\psi}$ and from the latter the equations satisfied by ψ'. The result is that the equations for ψ' can also be obtained directly from (1)—(6)— by the substitution

$$\psi \to \psi', \quad A_\mu \to A'_\mu. \tag{25}$$

Thus the substitution $\psi \to \psi'$ corresponds to the change of the sign of the electric charge. It changes the sign of the current J_μ and also of I_3. Therefore it changes also the signs of U and N.

The terms $L_{(N)}$, $L_{(\pi)}$ etc. in the total Lagrangian are invariant under the substitution (25). From the invariance of the term $L_{(strong)}$ under the reflections in the isobaric space and from (22a) we can see that $L_{(st)}$ is invariant under the substitution $\psi \to \psi'$, $\dot{\psi} \to \dot{\psi}'$. Thus the total Lagrangian is invariant under (25).

It can be shown that the total Lagrangian is invariant also under the change of the electromagnetic gauge and under two different constant-phase transformations (as shown by d'Espagnat and Prentki already). These invariances correspond to the conservation of Q, N and U.

The author would like to thank Prof. L. Rosenfeld for his kind and stimulating interest in this work.

Appendix

The infinitesimal rotation

$$f_i \equiv \psi^+ \zeta_i \psi \to (\delta_{ik}+\varepsilon_{ik}) f_k$$

of the isopseudovector **f** corresponds to the infinitesimal transformation

$$\psi \to (1+\tfrac{1}{2}\varepsilon_{ik}[\zeta_i, \zeta_k])\psi. \tag{1A}$$

Similarly the reflection

$$f_1 \to f_1,\ f_2 \to -f_2,\ f_3 \to -f_3$$

on the plane perpendicular to the 1-st axis corresponds to

$$\psi \to (1-2\zeta_1^2)\psi. \tag{2A}$$

This is easily verified by means of the commutation relations of the ζ's. The total reflection in the isobaric space then corresponds to $\psi \to \psi$ because $(1-2\zeta_1^2)\cdot(1-2\zeta_2^2)\cdot(1-2\zeta_3^2) = 1$ and the three factors are mutually commutative. From (10) and (1A), (2A) we obtain *the same* transformations also for $Z\psi^{+\mathrm{T}}$.

The "isobaric components" of our wave function $\psi_{(\Sigma)}$ are certain *linear combinations* of components $\chi_{(\Sigma)j}$ of an isopseudovector and conversely.

If we write

$$\chi_{(\Sigma)j} = D_j^+ \psi_{(\Sigma)} \tag{3A}$$

then the matrices D_j (with one column and three rows) fulfil the relations

$$\begin{gathered} D_j^+ D_k = \delta_{jk},\ D_j D_j^+ = 1, \\ D_j^+ \zeta_k D_l = i\varepsilon_{jkl}, \\ \zeta_j \zeta_k D_l = \delta_{jk} D_l - \delta_{jl} D_k, \\ \zeta_j D_k = -\zeta_k D_j,\ D_j^+ Z = D_j^{\mathrm{T}}. \end{gathered} \tag{4A}$$

The matrices D_j are the "isobaric" analogues of the Harish-Chandra [2]) $\mathcal{T}$-matrices (see below). The relations (3A), (4A) are invariant under the unitary transformations

$$\zeta_j \to u\zeta_j u^+, \quad Z = uZu^{\mathrm{T}}$$
$$D_j \to uD_j, \quad \psi_{(\Sigma)} \to u\psi_{(\Sigma)}, \quad \chi_{(\Sigma j)} \to \chi_{(\Sigma)j}$$

and can therefore be verified by means of a special representation of the ζ's.

Notice that with $\dot{\chi}_{(\Sigma)j} \equiv D_j^+ \dot{\psi}_{(\Sigma)}$ the equation $\dot{\psi}_{(\Sigma)} = CZ\bar{\psi}^{\mathrm{T}}_{(\Sigma)}$ can be transcribed $\dot{\chi}_{(\Sigma)j} = C\bar{\chi}^{\mathrm{T}}_{(\Sigma)j} = C\gamma_4^{\mathrm{T}}\chi^{+\mathrm{T}}_{(\Sigma)j}$.

The matrices β_μ fulfil the Duffin-Kemmer relations

$$\beta_\lambda\beta_\mu\beta_\nu + \beta_\nu\beta_\mu\beta_\lambda = \delta_{\lambda\mu}\beta_\nu + \delta_{\nu\mu}\beta_\lambda. \tag{5A}$$

The transformations of $\psi_{(\pi)}$ which correspond to the infinitesimal Lorentz transformation and to the total space-reflection are

$$\psi_{(\pi)} \to (1 + \tfrac{1}{2}\varepsilon_{\mu\nu}[\beta_\mu, \beta_\nu])\psi_{(\pi)} \tag{6A}$$

and

$$\psi_{(\pi)} \to (1 - 2\beta_4^2)\dot{\psi}_{(\pi)}. \tag{7A}$$

Then it can be found that $\dot{\psi}_{(\pi)}$ transforms in the same manner as $\psi_{(\pi)}$. The quantities $\bar{\psi}_{(\pi)}\beta_\mu\psi_{(\pi)}$ and $\bar{\psi}_{(\pi)}\psi_{(\pi)}$ behave as a true four-vector and true scalar respectively. Of course, both are isoscalars.

Now let $\mathcal{T}_\mu$ be the Harish-Chandra matrices which in the case of spin 0 have the following properties:

$$\begin{gathered}\mathcal{T}_\mu^+\mathcal{T}_\nu = \delta_{\mu\nu}, \quad \mathcal{T}_\nu\mathcal{T}_\nu^+ = \beta, \\ \beta^2 = \beta, \quad \beta_\nu\beta + \beta\beta_\nu = \beta_\nu, \\ \mathcal{T}_\lambda^+\beta_\mu\mathcal{T}_\nu = 0, \quad \beta_\lambda\beta_\mu\mathcal{T}_\nu = \mathcal{T}_\lambda\delta_{\lambda\nu}, \\ \beta_\mu\mathcal{T}_\nu = \tfrac{1}{4}\delta_{\mu\nu}\beta_\lambda\mathcal{T}_\lambda, \quad \mathcal{T}_\mu^+\mathrm{B} = -\mathcal{T}_\mu^{\mathrm{T}}.\end{gathered} \tag{8A}$$

Then it can be verified that the quantities

$$\varphi_{(\pi)\mu j} = \mathcal{T}_\mu^+ D_j^+ \psi_{(\pi)}$$

form a pseudo-four-vector and isopseudovector, whereas

$$\varphi_{(\pi)j} = \mathcal{T}_\mu^+\beta_\mu D_j^+\psi_{(\pi)}$$

form a pseudoscalar and isopseudovector, if besides (6A), (7A) also (1A) and (2A) are valid for $\psi_{(\pi)}$. With $\dot{\varphi}_{(\pi)j} = \mathcal{T}_\mu^+\beta_\mu D_j^+\dot{\psi}_{(\pi)}$ the equation $\dot{\psi}_{(\pi)} = BZ\bar{\psi}^{\mathrm{T}}_{(\pi)}$ can be transcribed as $\dot{\varphi}_{(\pi)j} = \varphi^+_{(\pi)j}$ so that the condition (18) gives $\varphi^+_{(\pi)j} = \varphi_{(\pi)j}$.

Assuming for the sake of simplicity that θ-mesons have spin 0 and that instead of (7A)

$$\psi_{(\theta)} \to -(1-2\beta_4^2)\psi_{(\theta)} \tag{9A}$$

is valid for total ordinary space reflection, we can form from $\psi_{(\theta)}$ the four-vector-isospinor

$$\vartheta_{(\theta)\mu} = T_\mu^+ \psi_{(\theta)}$$

and the scalar-isospinor

$$\vartheta_{(\theta)} = T_\mu^+ \beta_\mu \psi_{(\theta)}$$

which can be used in constructing the Lagrangian of "strong" interaction of θ-mesons with baryons.

References

1) B. d'Espagnat and J. Prentki, Nuc. Phys. **1** (1956) 33
2) Harish-Chandra, Proc. Roy. Soc. A **186** (1946) 502
3) M. Neumann and W. H. Furry, Phys. Rev. **76** (1949) 1677

BOOK REVIEWS

Interscience tracts on physics and astronomy (Interscience Publishers Ltd, New York, London. Each volume $ 1.75 or 14 s. with paper cover, $ 3.50 or 25 s. with hard cover).

This series of small, handy tracts (each of about 150 pages of 5¼ × 8 in), under the Editorship of Prof. R. E. Marshak, promises to fulfil an important part in the increasingly urgent task of bringing the rapidly accumulating knowledge in the various branches of physics and astrophysics within easy reach of harassed and overspecialised workers in these sciences, who find it more and more difficult to keep abreast of the general progress. The volumes published so far, at any rate, are models of clear and concise exposition and manage to condense a surprising amount of first-hand information in a small space. They can be said to provide on the one hand the best possible quick introduction into the subject for non-specialists, and on the other the most handy vade-mecum for specialists. Moreover, they will be quite serviceable to advanced students. The first two volumes, Dr. D. J. Hughes' *Neutron Optics* (1953) and Prof. M. S. Livingston's *High-energy accelerators* (1954) are already too well-known now to need any detailed review. The last volume to appear, Prof. L. Spitzer's *Physics of fully ionised gases* (1956) falls mostly outside the scope of this journal; however, it does contain information of the greatest value for the physics of cyclotrons and should not be overlooked by people interested in these problems. Editor and publishers, who are just announcing a fourth volume, must certainly be warmly congratulated and encouraged to pursue a most useful undertaking.

L. R.

H. A. Bethe and P. Morrison, *Elementary nuclear theory*, 2nd Edition (John Wiley & Sons, New York — Chapman & Hall, London 1956. xi—274 pages. $ 6.25).

On its first appearance in 1947 this book, in spite of its sketchy character, fulfilled a real service. But it may well be asked whether it was worth while to patch it up after those nine years which shook the world of nuclear physics. The old subdivisions of the subject are sprung or cracked and many questions appear in a different light altogether. It is a telling evidence of the pace of progress that the new sections added to this edition already show some signs of obsolescence. All the book can claim to be is a reasoned guide to more elaborate treatments presented in survey articles or other books.

L. R.

R. Evans, *The atomic nucleus* (McGraw-Hill Book Co., New York, Toronto, London 1955. xv—972 pages. 109 s.).

The statement in the preface that this volume, with its thousand closely printed pages 'represents the present content of a two-semester course in nuclear physics' must be taken with a grain of salt. It also contains 33 pages of references and a series of appendices of nearly 90 pages which constitute a well thought-out compendium of the general concepts, formulae and numerical data of constant use in nuclear physics. Moreover, the text is interspersed with a rich collection of well-chosen problems offered to the student's sagacity (some of them taken straight from published papers) and is so thorough and complete, with all possible graphs and tables of data, that it is almost an encyclopaedia of the subject. And yet the author has managed, with incredible skill, to avoid the pitfalls of such a treatment:

his exposition is so clear and neat that the reader's interest is kept alive and the abundance of details is not allowed to obscure the essential features.

The numerous sections (including a fascinating chapter on statistics) in which the experimental techniques are discussed with great care and thoroughness will make most instructive reading for theoretical physicists, while experimentalists in need of guidance on any matter of theory will be well-advised to look up the relevant section in this book. For the students, the book is not meant, of course, as a text to be mastered in its entirety, but as a self-sufficient repository of all the material needed for the further elaboration of the topics discussed in the lectures; they could not hope for a better companion.

L. R.

G. M. MURPHY (Editor), *Production of Heavy Water* (National Nuclear Energy Series; Division III, Volume 4 F).

(Mc Graw-Hill Book Co., New York, Toronto, London 1955. xvii—314 pages, 39 s.6 d.).

This book gives a detailed survey of the work on this subject carried out in the USA during World War II under the Manhattan Project of the Atomic Energy Commission. The reports from which it is composed have been written by various specialists in 1946, but have only be declassified and published in 1955.

Part I, concerning the commercial production of heavy water, is an expert survey of heavy water plants and their operation as they existed at that time.

Part II of the book considers the laboratory and pilot plant studies for the various processes investigated.

In the appendix of 35 pages many details about the subjects are given.

The following subjects are dealt with: water-distillation, electrolytic process, catalytic exchange between hydrogen and steam, some dual temperature processes (the dual temperature process which is the most important now, viz. that with steam and hydrogen sulfide, is not incorporated, however), and a very provisional study on hydrogen-distillation.

It is a well produced book, very lucidly and clearly written, in which the subjects are dealt with thoroughly.

Yet it is only of relative importance owing to the many drawbacks inherent in its scheme, especially the time-lag of about nine years between writing and publication of the material and the fact that nothing is said about the work done after 1946.

One should certainly not use the book to get a review of the possible processes for the production of heavy water and their relative economic merits. For this purpose the recent article by Manson Benedict in Proc. of Inst. Conf. on Atomic Energy, Geneva, Aug. 55, Vol. 8, pag. 377, should be taken.

For the specialist who wants to make a study of the processes, the book contains a wealth of information; this, however, with the exception of the hydrogen-distillation. This process, undoubtedly the cheapest for the production of heavy water, which will therefore be of great importance in the future, was at that time not developed owing to the uncertainty regarding the length of time required to solve the novel engineering problems connected with handling large quantities of gases at very low temperatures.

The above comments are not meant to criticize the authors, who have succeeded in providing a valuable contribution within the limited scope of the book.

P. J. Haringhuizen

NEWS ITEMS

NUCLEAR PHYSICS IN HUNGARY

Research in nuclear physics and cosmic radiation phenomena in Hungary disposes over a Central Research Institute, whose laboratories occupy a vast area in the wooded hills which extend West of the Danube at some distance from the city of Budapest. This Institute, founded in 1950, is still in the process of expanding its installations; at present, there are about 100 research workers in its various departments, which are briefly described below.

Besides the Central Institute, there is an active group of theoretical physicists under the leadership of Prof. Novobátzky at the Institute of Physics of Budapest University, founded by Roland Eötvös and now bearing his name †. Several of these theoretical physicists, who are working on nuclear problems, are also attached to the Central Institute.

At the Budapest Technical High School, part of the research work in the Physics Department under the leadership of Prof. Gombás concerns problems of nuclear structure, especially computations of binding energies and mass distributions.

CENTRAL RESEARCH INSTITUTE FOR PHYSICS

Director I. Kovács, Assistant Directors L. Jánossy and K. Simonyi. The Institute contains the following departments.

1) *Department of Spectroscopy* (I. Kovács; leaders of the sections: T. Mátrai, L. Láng and B. Vorsatz). A high resolving Rowland grating, 21 feet radius, resolving power 2×10^5. Various spectrographs and spark inductors.

2a) *Cosmic Ray Department.* Cosmic ray work led by E. Fenyves and A. Somogyi. Various counter arrangements for the study of extensive air showers and non-ionizing component. Investigation by L. Nagy on the Rossi transition effect has shown the Bothe-maxima to be all spurious. Investigations with cloud chamber. Emulsion group to start.

2b) *Cosmic Ray Department* (second group). Investigation of photo-electric effect and properties of photons led by Zs. Náray. Main work so far working out of a technique for the reliable counting of photons with electron multipliers. Photon coincidence experiment. Michelson-interferometer at small intensities.

2c) *Cosmic Ray Department* (theoretical group; leader G. Szamosi).

3) *Department for Electromagnetic Waves* (leader P. Faragó). Measurement of nuclear magnetic moments together with Maria Gécs and J. Mertz. Work on microwave spectroscopy. In preparation: experiment on the photon character of cm-waves.

4) *Department of Atomic Physics* (leader K. Simonyi). Construction of Van de Graaff generator and cascade generator together with I. Mérey. Already working a 1 MeV Van de Graaff and a 800 keV cascade generator. Advanced preparation of a 4 MeV pressure generator. Also mass spectrograph under construction. Investigations by J. Erő and L. Keszthelyi of angle-correlation of gamma-quanta. Mechanism of discharge at low pressure (G. Schmidt). Production of multiply charged ions. Age determinations with C^{14} are in preparation together with the Cosmic Ray Group.

5) *Department of Radiology* (leader L. Bozóky; main co-worker O. Haimann). Construction of instruments and detection of radiation. A. Szilvássy: X-ray dosimetry

† The experimental work at the Roland Eötvös Institute (Prof. Gyulyai) is mainly concentrated on crystal physics.

and effects of X-rays, in collaboration with L. Bozóky. Hot laboratory prepares strong radioactive substances for the Institute and other institutions.

6) *Department of Ferromagnetism* (leader L. Pál). Experimental and theoretical investigations of the anisotropy of ferromagnetic semi-conductors. D. Pallagi carried out measurements on samples under the influence of 1000 to 3000 MHz. Solenoids with great homogeneity and also arrangements for the production of single crystals, constructed with the assistance of Gy. Zsigmond.

7) *Central Workshop*. To assist the scientific departments there is a Central Workshop supplying such needs which cannot be supplied by the departmental workshops.

An Electronic Group under the leadership of K. Zsdánszky constructs, in close collaboration with the departments, specialized electronic equipments.

FORD FOUNDATION GRANT TO CERN

CERN has recently accepted an offer by the Ford Foundation of a grant of $ 400 000, to be spent over five years, to help in strengthening co-operation in nuclear physics research, primarily with the United States and with other countries not members of CERN.

The grant is expected to be used mainly to enable Guest Professors to come and work with CERN, and to give young scientists opportunities of working in its laboratories.

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Compiled by K. Way, R. W. King, C. L. McGinnis, R. van Lieshout
Nuclear Data Group, U.S.A. National Academy of Sciences-National Research Council - Washington 25, D.C.

Publication TID-5300 of the U.S.A. Atomic Energy Commission (1955)

Nuclear Level Schemes is designed for the nuclear scientist and technologist.

It contains 53 half-page diagrams of decay schemes showing positions and properties of nuclear energy levels, characteristics of radioactive disintegrations, capture gamma rays, neutron and proton binding energies, etc.

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The inclusion of private communications regarding work in press makes the book up-to-date as of the end of 1955 although the literature cut-off date is June 1955.

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